

Radiometry and Photometry

8.1 Introduction

In concept, both radiometry and photometry are quite straightforward; however, both have been cursed with a jungle of often bewildering terminology. Radiometry deals with radiant energy (i.e., electromagnetic radiation) of any wavelength. Photometry is restricted to radiation in the visible region of the spectrum. The basic unit of power (i.e., rate of transfer of energy) in radiometry is the watt; in photometry, the corresponding unit is the lumen, which is simply radiant power as modified by the relative spectral sensitivity of the eye (Fig. 5.10) per Eq. 8.18. Note that watts and lumens have the same dimensions, namely energy per time.

All radiometry must take into account the variation of characteristics with wavelength. Examples are the spectral variation of emission, the variation of transmission of the atmosphere and optics with wavelength, and the differences in detector and film response with wavelength. A convenient way to deal with this is to multiply, wavelength by wavelength, all such factors together so as to arrive at one unified spectral weighting function. Thus, all radiometry is spectrally weighted and it should be apparent that photometry is simply one particular spectral weighting. See Sec. 8.9.

The principles of radiometry and photometry are readily understood when one thinks in terms of the basic units involved, rather than the special terminology which is conventionally used. The next five sections will discuss radiation in terms of watts; the reader should

remember that the discussion is equally valid for photometry, if lumens are read for watts.

8.2 The Inverse Square Law; Intensity

Consider a hypothetical point (or “sufficiently” small) source of radiant energy, which is radiating uniformly in all directions. If the rate at which energy is radiated is P watts, then the source has a radiant intensity J of $P/4\pi$ watts per steradian,* since the solid angle into which the energy is radiated is a sphere of 4π steradians. Of course there are no truly “point” sources and no practical sources which radiate uniformly in all directions, but if a source is quite small relative to its distance, it can be treated as a point, and its radiation, in the directions in which it does radiate, can be expressed in watts per steradian.

If we now consider a surface which is S cm from the source, then 1 cm² of this surface will subtend $1/S^2$ steradians from the source (at the point where the normal from the source to the surface intersects the surface, if S is large). The irradiance H on this surface is the incident radiant power per unit area and is obtained by multiplying the intensity of the source in watts per steradian by the solid angle subtended by the unit area. Thus, the irradiance is given by

$$H = J \frac{1}{S^2} \quad (8.1)$$

The units of irradiance are watts per square centimeter (W/cm²). Equation 8.1 is, of course, the “inverse square” law, which is conventionally stated: the illumination (irradiance) on a surface is inversely proportional to the square of the distance from the (point) source.

Thus, if our uniformly radiating point source emits energy at a rate of 10 W, it will have an intensity $J = 10/4\pi = 0.8$ W ster⁻¹, and the radiation falling on a surface 100 cm away would be 0.8×10^{-4} W/cm², or 80 μW/cm². If the surface is flat, the irradiance will, of course, be less than this at points where the radiation is incident at an angle, since the solid angle subtended by a unit of area in the surface will be reduced. From Fig. 8.1 it can be seen that the source-to-surface distance is increased to $S/\cos \theta$ and that the effective area (normal to the

*A steradian is the solid angle subtended (from its center) by $1/4\pi$ of the surface area of a sphere. Thus, a sphere subtends 4π (12.566) steradians from its center; a hemisphere subtends 2π steradians. The size of a solid angle in steradians is found by determining the area of that portion of the surface of a sphere which is included within the solid angle and dividing this area by the square of the radius of the sphere. For a small solid angle, the area of the included flat surface normal to the “central axis” of the angle can be divided by the square of the distance from the surface to the apex of the angle to determine its size in steradians. One can visualize a steradian as a cone with an apex angle of about 65.5°, or 3283 square degrees.

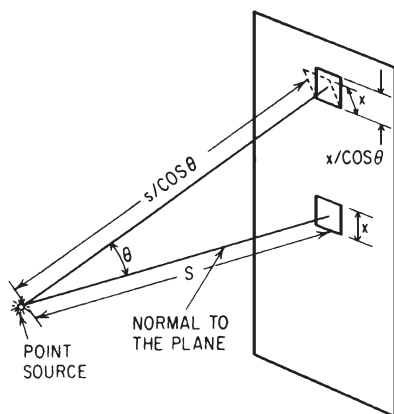


Figure 8.1 Geometry of a point source irradiating a plane, showing that irradiance (or illumination) varies with $\cos^3 \theta$.

direction of the radiation) is reduced by a $\cos \theta$ factor. Thus, the solid angle subtended, and the irradiance, are reduced by a $\cos^3 \theta$ factor.

8.3 Radiance and Lambert's Law

An extended source, that is, one whose dimensions are significant, must be treated differently than a point source. A small area of the source will radiate a certain amount of power per unit of solid angle. Thus, the radiation characteristics of an extended source are expressed in terms of power per unit solid angle per unit area. This is called *radiance*; the usual units for radiance are watts per steradian per square centimeter ($\text{W ster}^{-1} \text{cm}^{-2}$) and the symbol is N . Note that the area is measured normal to the direction of radiation, not in the radiating surface.

Most extended sources of radiation follow, at least approximately, what is known as Lambert's law of intensity,

$$J_{\theta} = J_0 \cos \theta \quad (8.2)$$

where J_{θ} is the intensity of a small incremental area of the source in a direction at an angle θ from the normal to the surface, and J_0 is the intensity of the incremental area in the direction of the normal. For example, a heated metal disk with a total area of 1 cm^2 and a radiance of $1 \text{ W ster}^{-1} \text{cm}^{-2}$ will radiate 1 W/ster in a direction normal to its surface. In a direction 45° to the normal, it will radiate only 0.707 W/ster ($\cos 45^\circ = 0.707$).

Notice that although radiance is given in terms of watts per steradian per square centimeter, this should not be taken to mean that the radiation is uniform over a full steradian or over a full square centimeter. Consider a source consisting of a 0.1-cm square incandescent

filament in a 20-cm-diameter envelope. Assume that the bulb is painted so that only a 1-cm square transmits energy, and that the source radiates one-fiftieth of a watt through this square. (We assume, for convenience, that the radiation intercepted by the painted envelope is thereby totally removed from consideration.) Now the filament has an area of 0.01 cm^2 and is radiating 0.02 W into a solid angle of (approximately) 0.01 steradian . Therefore, it has a radiance of $200 \text{ W ster}^{-1} \text{ cm}^{-2}$, but only within the solid angle subtended by the window! Outside this angle the radiance is zero. This concept of radiance over a limited angle becomes important in dealing with the radiance of images and must be thoroughly understood.

There are several interesting consequences of Lambert's law that are worthy of consideration, not only for their own sake but because they illustrate the basic techniques of radiometric calculations. *The radiance of a surface is conventionally taken with respect to the area of a surface normal to the direction of radiation.* It can be seen that, although the emitted radiation per steradian falls off with $\cos \theta$ according to Lambert's law, the "projected" surface area falls off at exactly the same rate. The result is that *the radiance of a Lambertian surface is constant with respect to θ .* In visual work the quantity corresponding to radiance is brightness, and the above is readily demonstrated by observing that the brightness of a diffuse source is the same regardless of the angle from which it is viewed.

8.4 Radiation into a Hemisphere

Let us determine the total power radiated from a flat diffuse source into a hemisphere. If the source has a radiance of $N \text{ W ster}^{-1} \text{ cm}^{-2}$, one might expect that the power radiated into a hemisphere of 2π steradians would be $2\pi N \text{ W/cm}^2$. That this is twice too large is readily shown. With reference to Fig. 8.2, let A represent the area of a small source with a radiance of $N \text{ W ster}^{-1} \text{ cm}^{-2}$ and an intensity of $J_\theta = J_0 \cos \theta = NA \cos \theta \text{ W/ster}$. The incremental ring area on a hemisphere of radius R has an area of $2\pi R \sin \theta \cdot R d\theta$ and thus subtends (from A) a solid angle of $2\pi R^2 \sin \theta d\theta / R^2 = 2\pi \sin \theta d\theta$ steradians. The radiation intercepted by this ring is the product of the intensity of the source and the solid angle, or

$$dP = J_\theta 2\pi \sin \theta d\theta = 2\pi NA \sin \theta \cos \theta d\theta \quad (8.3)$$

Integrating to find the total power radiated into the hemisphere from A , we get

$$P = \int_0^{\pi/2} 2\pi NA \sin \theta \cos \theta d\theta = 2\pi NA \left[\frac{\sin^2 \theta}{2} \right]_0^{\pi/2} = \pi NA \text{ watts} \quad (8.4)$$

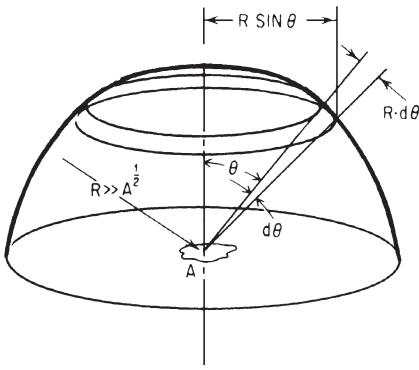


Figure 8.2 Geometry of a Lambertian source radiating into a hemisphere.

Dividing by A to get watts emitted per square centimeter of source, we find the radiation into the 2π steradian of the hemisphere to be πN W/cm², not $2\pi N$. This is the basic relationship between radiance and the power emitted from the surface.

8.5 Irradiance Produced by a Diffuse Source

It is frequently of interest to determine the irradiance produced at a point by a Lambertian source of finite size. Referring to Fig. 8.3, assume that the source is a circular disk of radius R and that we wish to determine the irradiance at some point X which is a distance S from the source and is on the normal through the center of the source. (Note that we will determine the irradiance on a plane parallel to the plane of the source.) The radiant intensity of a small element of area dA in the direction of point X is given by Eq. 8.2 as

$$J_{\theta} = J_0 \cos \theta = N dA \cos \theta$$

where N is the radiance of the source. Since the distance from dA to X is $S/\cos \theta$, and the radiation arrives at an angle θ , the incremental irradiance at X produced by dA is

$$dH = J_0 \cos \theta \left[\frac{\cos^3 \theta}{S^2} \right] = \frac{N dA \cos^4 \theta}{S^2} \quad (8.5)$$

The same irradiance is produced by each incremental area making up a ring of radius r and a width dr , so that we can substitute the area of the ring, $2\pi r dr$, for dA in Eq. 8.5 to get the incremental irradiance from the ring.

$$dH = \frac{2\pi r dr N \cos^4 \theta}{S^2} \quad (8.6)$$

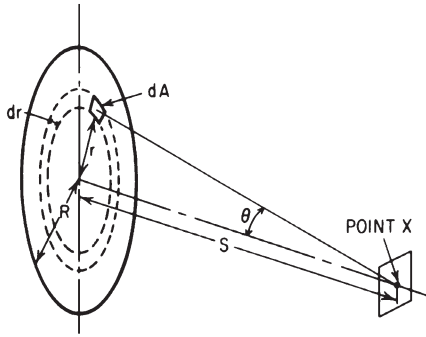


Figure 8.3 Geometry of a circular source irradiating point X.

To simplify the integration, we substitute

$$r = S \tan \theta$$

$$dr = S \sec^2 \theta d\theta$$

into Eq. 8.6 to get

$$dH = \frac{2\pi S \tan \theta S \sec^2 \theta d\theta N \cos^4 \theta}{S^2}$$

$$= 2\pi N \tan \theta \cos^2 \theta d\theta = 2\pi N \sin \theta \cos \theta d\theta$$

Integrating to determine the irradiance from the entire source, we get

$$H = \int_0^{\theta_m} 2\pi N \sin \theta \cos \theta d\theta = 2\pi N \left[\frac{\sin^2 \theta}{2} \right]_0^{\theta_m}$$

$$H = \pi N \sin^2 \theta_m \text{ watt/cm}^2 \quad (8.7)$$

where H is the irradiance produced at a point by a circular source of radiance $N \text{ W ster}^{-1} \text{ cm}^{-2}$ which subtends an angle of $2\theta_m$ from the point (when the point is on the “axis” of the source). Note well that θ_m is the angle defined by the source diameter.

Unfortunately noncircular sources do not readily yield to analysis. However, *small* noncircular sources may be approximated with a fair degree of accuracy by noting that the solid angle subtended by the source from X is

$$\Omega = 2\pi (1 - \cos \theta) = 2\pi \frac{\sin^2 \theta}{(1 + \cos \theta)}$$

and for small values of θ , $\cos \theta$ approaches unity and

$$\omega = \pi \sin^2 \theta$$

Thus, if the angle subtended by the source is moderate, we can substitute into Eq. 8.7 and write

$$H = N\omega \quad (8.8)$$

If the point X does not lie on the “axis” (the normal through the center of the circular source), then the irradiance would be subject to the same factors outlined in the discussion of the “cosine-fourth” rule in Sec. 6.7. Thus, if the line from the point X_ϕ to the center of the circle makes an angle ϕ to the normal, the irradiance at X_ϕ is given by

$$H_\phi = H_0 \cos^4 \phi \quad (8.9)$$

where H_0 is the irradiance along the normal given by Eq. 8.7 or 8.8 and H_ϕ is the irradiance at X_ϕ (measured in a plane parallel to the source). (See the note in Example A regarding the inaccuracy of the cosine-fourth rule when the angles θ and ϕ are large.)

It is apparent that Eqs. 8.8 and 8.9 may be used in combination to calculate the irradiance produced by any conceivable source configuration, to whatever degree of accuracy that time (or patience) allows.

8.6 The Radiometry of Images; The Conservation of Radiance

When a source is imaged by an optical system, the image has a radiance, and it may be treated as a secondary source of radiation. However, one must always keep in mind that the radiance of an image differs from the radiance of an ordinary source in that the radiance of an image exists *only* within the solid angle subtended from the image by the clear aperture of the optical system. Outside of this angle, the radiance of the image is zero.

Figure 8.4 illustrates an aplanatic optical system imaging an incremental area A of a lambertian source at A' . We will consider the radiance of the image at A' formed through a generalized incremental area P in the principal surface of the optical system. (Since the system is aplanatic, that is, free of coma and spherical aberration, the principal “planes” are spherical surfaces and are centered on the object and image.) The radiance of the source is N W ster⁻¹ cm⁻² and the projected area of A in the direction θ is $A \cos \theta$ cm². The solid angle subtended by incremental area P from A is P/S^2 , where S is the distance from the object to the first principal surface. Therefore, the radiant power intercepted by area P is

$$\text{Power} = N \frac{P}{S^2} A \cos \theta \text{ watts}$$

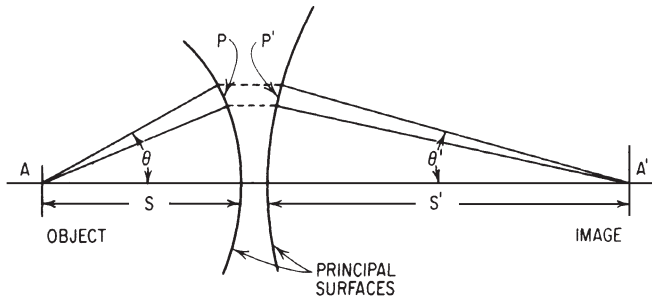


Figure 8.4 Illustrates an aplanatic optical system imaging an incremental source area A at A' .

This radiation is imaged by the optical system at area A' , into a (projected) area $A' \cos \theta'$, through a solid angle P'/S'^2 . Thus, the radiance at A' is given by

$$N' = TN \frac{P}{S^2} A \cos \theta \left[\frac{S'^2}{P'A' \cos \theta'} \right] \text{ watt ster}^{-1} \text{ cm}^{-2}$$

where T is the transmission of the optical system. Now we note that the incremental areas A and A' are related by the laws of first-order optics, and, if both are in media of the same index, $AS'^2 = A'S^2$. Further, the principal surfaces are unit images of each other; taking the tilts of the surfaces into account, we get $P \cos \theta = P' \cos \theta'$. Making these substitutions and clearing, we find that *the radiance of the image is equal to that of the object times the transmission of the system, or*

$$N' = TN \tag{8.10}$$

This fundamental relationship can be restated with slightly different emphasis: the radiance of an image cannot exceed that of the object.*

*This statement and Eqs. 8.10 and 8.11 are subject to the condition that both object and image lie in media of the same index of refraction. When the media have different indices, the image radiance and irradiance are multiplied by the factor $(n_i/n_0)^2$, where n_i and n_0 are the refractive indices of the image media and object media, respectively. Thus, Eqs. 8.10 and 8.11 become

$$N' = TN \left(\frac{n_i}{n_0} \right)^2 \tag{8.10a}$$

$$\begin{aligned} H &= TN \left(\frac{n_i}{n_0} \right)^2 \pi \sin^2 \theta' \\ &= TN \left(\frac{n_i}{n_0} \right)^2 \omega \end{aligned} \tag{8.11a}$$

The factor (n_i/n_0) is introduced by the use of $An_0^2S'^2 = A'n_i^2S^2$ in place of $AS'^2 = A'S^2$ in the derivation of Eq. 8.10; both equalities are derived from the optical invariant relationship $hnu = h'n'u'$ (Eq. 2.55).

At first consideration the *conservation of radiance* (or *brightness*) seems quite counterintuitive. Ordinarily, the solid angle of radiation accepted by an optical system from a source is quite small, as is the fraction of the total power which passes through the lens and forms the image. It is difficult to accept that the image formed by this small fraction of the source power will have the same radiance as does the source. We can easily demonstrate this, using only the first-order optics from Chap. 2.

Let us assume a small source of radiance N with an area A . The source thus has an intensity of AN . The source is imaged by an optical system with an area P which is located a distance S from the source. The solid angle subtended by the lens from the source is thus P/S^2 , and the power intercepted by the lens and formed into the image is ANP/S^2 .

The lens will form an image with a magnification M , and the area of the image will thus be AM^2 . The image distance will be MS , and the solid angle subtended by the lens from the image will be P/M^2S^2 ster. Thus the power in the image (ANP/S^2) is spread over the image area (AM^2) and exists only over the solid angle (P/M^2S^2). The image radiance is power per unit area per solid angle; combining the expressions above, we get (neglecting any transmission losses)

$$\begin{aligned}\text{Image radiance} &= \text{power/area} \cdot \text{solid angle} \\ &= (ANP/S^2) / (AM^2) (P/M^2S^2)\end{aligned}$$

We can cancel A , P , S , and M , leaving us with

$$\text{Image radiance} = N \text{ (the object radiance)}$$

which is a statement of the *conservation of radiance* (or *brightness*).

By the application of exactly the same integration technique used in Sec. 8.5, it can be shown that the *irradiance* produced in the plane of an image is given by

$$H = T\pi N \sin^2 \theta' \text{ watt/cm}^{-2} = TN\omega \text{ (for small angles)} \quad (8.11)$$

where T is the system transmission, N ($\text{W ster}^{-1} \text{ cm}^{-2}$) is the object radiance, and θ' is the half angle subtended by the exit pupil of the optical system from the image. Small or noncircular exit pupils and cylindrical lens systems can be handled by substituting the solid angle ω for $\pi \sin^2 \theta'$ (just as in Eq. 8.8); image points off the optical axis are subject to the cosine-fourth law in addition to any losses due to vignetting (Eq. 8.9 and Sec. 6.7).

The similarity between the equations for the irradiance produced by a diffuse source and by an optical system makes it apparent that, when it is viewed from the image point, the aperture of the optical system takes on the radiance of the object it is imaging. This is an

extremely useful concept; for radiometric purposes, a complex optical system can often be treated as if it consisted solely of a transmission loss and an exit pupil with the same radiance as the object. Similarly, when an optical system produces an image of a source, the image can be treated as a new source of the same radiance (less transmission losses). Of course, the direction that radiation is emitted from the image is limited by the aperture of the system.

When an object is so small that its image is a diffraction pattern (Airy disk), then the preceding techniques, which apply to extended sources, cannot be used. Instead, the power intercepted by the optical system, reduced by transmission losses, is spread into the diffraction pattern. To determine the irradiance (or the radiance) of the image, we note that 84 percent of the power intercepted and transmitted by the lens is concentrated into the central bright spot (the Airy disk). A precise determination of irradiance requires that one integrate the relative irradiance-times-area product over the central disk and equate this to 84 percent of the image power. If P is the total power in the Airy pattern, H_0 the irradiance at the center of the pattern, and z the radius of the first dark ring, a numerical integration of Eq. 6.18 over the central disk yields

$$0.84P = 0.72H_0z^2$$

Rearranging and substituting the value of z given by Eq. 6.20, we get

$$H_0 = 1.17 \frac{P}{z^2} = \pi P \left(\frac{\text{NA}}{\lambda} \right)^2$$

where λ is the wavelength and NA is $n' \sin U'$, the numerical aperture. The irradiance for points not at the center of the pattern is then found by Eq. 6.18. Note that the preceding assumes a circular aperture; for rectangular apertures, the process would be based on Eq. 6.16.

Example A

In Fig. 8.5, A is a circular source with a radiance of 10 W per ster per cm^2 radiating toward plane BC . The diameter of A subtends 60° from point B . The distance AB is 100 cm and the distance BC is 100 cm. An optical system at D forms an image of the region about point C at E . Plane BC is a diffuse (lambertian) reflector with a reflectivity of 70 percent. The optical system (D) has a 1-in-square aperture and the distance from D to E is 100 in. The transmission of the optical system is 80 percent. We wish to determine the power incident on a 1-cm square photodetector at E .

We begin by determining the irradiance at B , using Eq. 8.7; the source radiance is $10 \text{ W ster}^{-1} \text{ cm}^{-2}$ and the half angle θ is 30° , giving

$$H_B = \pi N \sin^2 \theta = \pi \cdot 10 \cdot \left(\frac{1}{2} \right)^2 = 7.85 \text{ W/cm}^2$$

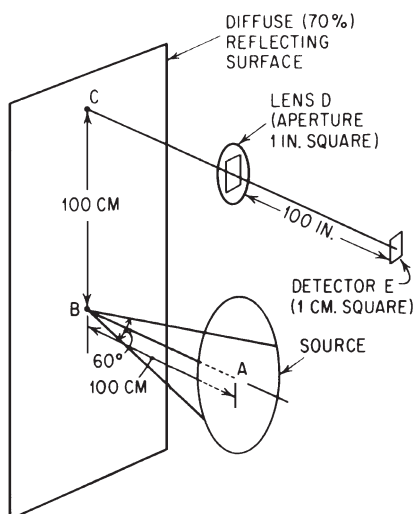


Figure 8.5 Example A.

Since angle BAC is 45° , we can find the irradiance at C from Eq. 8.9, noting that $\cos 45^\circ$ is 0.707

$$H_c = H_b \cos^4 45^\circ = 7.85 \times (0.707)^4 = 1.96 \text{ W/cm}^2$$

(Note that the cosine-fourth effect derived in Sec. 6.7 included one cosine term which was approximate; its accuracy depended on the distance from the pupil to the image surface being *much* larger than the pupil diameter. In Example A this approximation is quite poor. P. Foote, in the *Bulletin of the Bureau of Standards* 12, 583 (1915), gave the following expression for the irradiance, which is accurate even when the source is large compared with the distance.

$$H = \frac{\pi N}{2} \left[1 - \frac{(1 + \tan^2 \phi - \tan^2 \theta)}{[\tan^4 \phi + 2 \tan^2 \phi(1 - \tan^2 \theta) + 1/\cos^4 \theta]^{1/2}} \right]$$

If we compare the irradiance from this equation with that from Eqs. 8.7 and 8.8 for the angles ϕ and θ from Example A, we find that this irradiance is 42 percent greater than the cosine-fourth result. This is, of course, a rather extreme case.)

It is now necessary to determine the radiance of the surface at C . The diffuse surface at C reradiates 70 percent of the incident 1.96 W/cm^2 into a full hemisphere; the total power reradiated is thus 1.37 W/cm^2 . In Sec. 8.4 it was shown that a source of radiance N radiated $\pi N \text{ W/cm}^2$ into a hemisphere. Thus the radiance at point C is given by

$$N_C = \frac{RH}{\pi} = \frac{0.7 \times 1.96}{\pi} = \frac{1.37}{\pi} = 0.44 \text{ W ster}^{-1} \text{ cm}^{-2}$$

The irradiance at E can now be determined from Eq. 8.11, noting that the solid angle subtended by the aperture of the lens system is $1/(100)^2$, or 10^{-4} ster, and substituting this for $\pi \sin^2 \theta$ in Eq. 8.11,

$$\begin{aligned} H_E &= T_D \pi N_C \sin^2 \theta = T_D N_C \omega \\ &= 0.8 \times 0.44 \times 10^{-4} = 0.35 \times 10^{-4} \text{ W/cm}^2 \end{aligned}$$

Since the photodetector at E has an area of 1 cm^2 , the radiant power falling on it is just $0.35 \times 10^{-4} \text{ W}$, or $35 \mu\text{W}$.

8.7 Spectral Radiometry

In the preceding discussion, no mention has been made of the spectral characteristics of the radiation. It is apparent that every radiant source has some sort of spectral distribution of its radiation, in that it will emit more radiation at certain wavelengths than others.

For many purposes, it is necessary to treat intensity (J), irradiance (H), radiance (N), etc. (in fact, all the quantities listed in Fig. 8.6) as functions of wavelength. To do this we refer to the above quantities per unit interval of wavelength. Thus, if a source emits 5 W of radiant power in the spectral band between 2 and $2.1 \mu\text{m}$, it emits 50 W per micrometer ($\text{W}/\mu\text{m}$) in this region of the spectrum. The standard symbol for this type of quantity is the symbol given in Fig. 8.6 subscripted with a λ , and the name is preceded by "spectral." For example, the symbol for spectral radiance is N_λ and its units are watts per steradian per square centimeter per micrometer ($\text{W ster}^{-1} \text{ cm}^{-2} \mu\text{m}^{-1}$).

Name	Symbol	Description	Units
Radiant power (flux)	P (ϕ)	Rate of transfer of energy	W (Joule/sec)
Radiant intensity	J (I)	Power per unit solid angle from a source	W/ster ¹
Radiance	N (L)	Power per unit solid angle per unit area from a source	W ster ⁻¹ cm ⁻²
Irradiance	H (E)	Power per unit area incident on a surface	W/cm ²
Radiant energy	U		Joule
Radiant emittance	W (M)	Power per unit area emitted from a surface	W/cm ²

Figure 8.6 Radiometric terminology. The names, symbols, descriptions, and preferred units for quantities in radiometric work.

In many applications it is absolutely necessary to take the spectral characteristics of sources, detectors, optical systems, filters, and the like into account. This is accomplished by integrating the particular radiation product function over an appropriate wavelength interval. Since most spectral characteristics are not ordinary functions, the process of integration is usually numerical, and thus laborious. As a brief example, suppose that the irradiance in an image is desired. The spectral radiance of the object can be described by some function $N(\lambda)$ and the transmission of the atmosphere, the optical system, and any filters can be combined in a spectral transmission function $T(\lambda)$. Equation 8.11 will give the irradiance of the image (for any given wavelength); for use over an extended wavelength interval, we must write

$$H = \int_{\lambda_1}^{\lambda_2} T(\lambda) \pi N(\lambda) \sin^2 \theta d\lambda = \pi \sin^2 \theta \int_{\lambda_1}^{\lambda_2} T(\lambda) N(\lambda) d\lambda \text{ W/cm}^2 \quad (8.12)$$

where λ_1 and λ_2 , the limits of the integration, may be zero and infinity, but are usually taken as real wavelengths which encompass the region of interest. In practice, it is usually necessary to perform the integration numerically; this process is represented (for this particular example) by the summation:

$$H = \pi \sin^2 \theta \sum_{\lambda=\lambda_1}^{\lambda_2} T(\lambda) N(\lambda) \Delta\lambda \text{ W/cm}^2 \quad (8.13)$$

The spectral response of a detector is included in a calculation in the same manner. For example, the effective power falling on a detector with an area of A and a relative spectral response $R(\lambda)$, when the detector is located in the image plane of the system above, would be (provided that the image completely covered the detector)

$$P = A \pi \sin^2 \theta \int_{\lambda_1}^{\lambda_2} R(\lambda) T(\lambda) N(\lambda) d\lambda \text{ W}$$

8.8 Blackbody Radiation

A perfect blackbody is one which totally absorbs all radiation incident upon it. The radiation characteristics of a heated blackbody are subject to known laws, and since it is possible to build a close approximation to an ideal blackbody, a device of this type is a very useful standard source for the calibration and testing of radiometric instruments. Further, most sources of thermal radiation, i.e., sources which radiate because they are heated, radiate energy in a manner which can be readily described in terms of a blackbody emitting through a filter, making it possible to use the blackbody radiation laws as a starting point for many radiometric calculations.

Planck's law describes the spectral radiant emittance of a perfect blackbody as a function of its temperature and the wavelength of the emitted radiation.

$$W_{\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \quad (8.14)$$

where W_{λ} = the radiation emitted into a hemisphere by the blackbody in power per unit area per wavelength interval ($\text{W cm}^{-2} \mu\text{m}^{-1}$)

λ = the wavelength (μm)

e = the base of natural logarithms (2.718...)

T = the temperature of the blackbody in Kelvin ($\text{K} = ^{\circ}\text{C} + 273$)

C_1 = a constant = 3.742×10^4 when area is in square centimeters and wavelength in micrometers

C_2 = a constant = 1.4388×10^4 when square centimeters and micrometers are used

Figure 8.7 indicates the shape of the curve of W_{λ} plotted against wavelength. Note that the spectral radiance (N_{λ}) is given by W_{λ}/π .

If we integrate Eq. 8.14, we can obtain the total radiation at all wavelengths. The resulting equation is known as the *Stefan-Boltzmann law*,

$$W_{\text{TOT}} = 5.67 \times 10^{-12} T^4 \text{ W/cm}^2 \quad (8.15)$$

and indicates that the total power radiated from a blackbody varies as the fourth power of the absolute temperature.

If we differentiate Planck's equation (8.14) and set the result equal to zero, we can determine the wavelength at which the spectral emittance (W_{λ}) is a maximum and also the amount of W_{λ} at this wavelength. *Wien's displacement law* gives the wavelength for maximum W_{λ} as

$$\lambda_{\text{max}} = 2897.8 T^{-1} \mu\text{m} \quad (8.16)$$

and W_{λ} at λ_{max} as

$$W_{\lambda, \text{max}} = 1.286 \times 10^{-15} T^5 \text{ W/cm}^2 \cdot \mu\text{m}^{-1} \quad (8.17)$$

Notice that the higher the temperature, the shorter the wavelength at which the peak occurs and that W_{λ} at the peak varies as the fifth power of the absolute temperature.

Before the advent of the electronic calculator, Planck's equation was very awkward to use and for this reason a number of tables, charts,

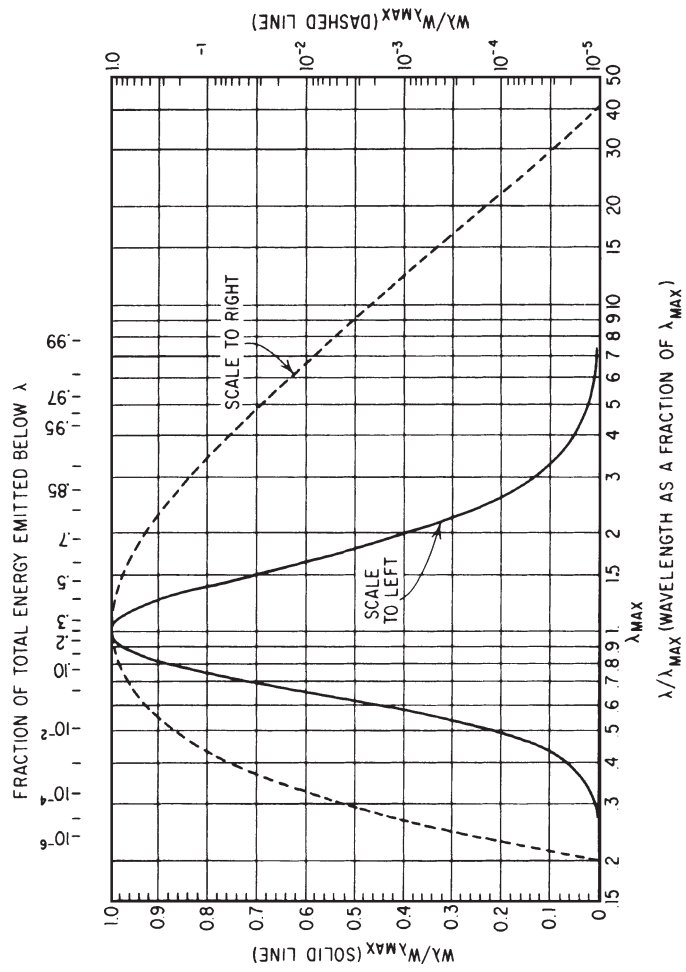


Figure 8.7 Spectral distribution of blackbody radiation (normalized).

and slide rules are available which allow the user to simply look up the values of W_λ for the appropriate temperature and wavelength. Figure 8.7 may be used for this purpose when the precision required is relatively modest.

The use of Fig. 8.7 is quite simple: First the total energy (W_{TOT}), the peak wavelength (λ_{max}), and the maximum spectral radiant emittance ($W_{\lambda, max}$) are calculated for the desired temperature by Eqs. 8.15, 8.16, and 8.17, respectively. The graph in Fig. 8.7 is of $W_\lambda/W_{\lambda, max}$ plotted against relative wavelength. Thus, if W_λ for a particular wavelength (λ) is desired, the value of $W_\lambda/W_{\lambda, max}$ corresponding to the appropriate value of λ/λ_{max} is selected and multiplied by the value of $W_{\lambda, max}$ from Eq. 8.17.

Across the top of Fig. 8.7 is a scale which indicates the fraction of the total energy emitted at all wavelengths below that corresponding to the point on the scale. Note that exactly 25 percent of the energy from a blackbody is emitted at wavelengths shorter than λ_{max} . If it is necessary to determine the amount of power emitted in a spectral band between two wavelengths (λ_1 and λ_2), the wavelengths are converted to relative wavelengths (λ_1/λ_{max} and λ_2/λ_{max}) and the fractions corresponding to them are selected from the scale at the top of the figure. The total power (W_{TOT}) from Eq. 8.15 times the difference between the two fractions will give the amount of power emitted in the wavelength interval.

Example B

For a blackbody at a temperature of 27°C (80.6°F), T is $273 + 27 = 300$ K, and the total emitted radiation is given by Eq. 8.15

$$W_{TOT} = 5.67 \times 10^{-12}(300)^4 = 4.59 \times 10^{-2} \text{ W/cm}^2$$

The wavelength at which W_λ is a maximum is given by Eq. 8.16

$$\lambda_{max} = 2897.9 (300)^{-1} = 9.66 \text{ } \mu\text{m}$$

and the radiant emittance at this wavelength is obtained from Eq. 8.17

$$W_{\lambda, max} = 1.288 \times 10^{-15} (300)^5 = 3.13 \times 10^{-3} \text{ W cm}^{-2} \mu\text{m}^{-1}$$

As an aside, note that this (300 K) is a reasonable value for the ambient temperature and that our result indicates that the earth and most things on it are strongly emitting at a wavelength of 10 μm . This is the basis of the “see in the dark” FLIR systems which are sensitive to this spectral region; most such systems use germanium optics, which transmit well in the 8- to 14- μm region (which also happens to

be a good transmission window of the atmosphere). Thus there is no such thing as darkness if you can detect 10- μm radiation.

Suppose we wish to know the characteristics of this blackbody in the wavelength region between 4 and 5 μm . We express these wavelengths in terms of λ_{max} as $4/9.66 = 0.414$ and $5/9.66 = 0.518$. From Fig. 8.7, the corresponding values of $W_\lambda/W_{\lambda, \text{max}}$ are 0.07 and 0.25; these values, multiplied by $W_{\lambda, \text{max}} = 3.13 \times 10^{-3} \text{ W cm}^{-2} \mu\text{m}^{-1}$ give us the spectral radiant emittances for these wavelengths

At 4 μm :

$$W_\lambda = 0.22 \times 10^{-3} \text{ W cm}^{-2} \mu\text{m}^{-1}$$

At 5 μm :

$$W_\lambda = 0.78 \times 10^{-3} \text{ W cm}^{-2} \mu\text{m}^{-1}$$

Using the fraction scale across the top of the chart, we find that about 0.011 of the radiation is emitted below 5 μm (rel. $\lambda = 0.518$) and about 0.0015 below 4 μm . Thus, approximately 1 percent of the total radiation (W_{TOT}), amounting to about $4 \times 10^{-4} \text{ W/cm}^2$, is emitted in this spectral band. The radiance of the surface will be $4 \times 10^{-4}/\pi \text{ W ster}^{-1} \text{ cm}^{-2}$ in this spectral band. If the blackbody is a foot square, with an area of about 1000 cm^2 , it will radiate about 0.4 W between 4 and 5 μm into a hemisphere of 2π ster.

Most thermal radiators are not perfect blackbodies. Many are what are called gray-bodies. A gray-body is one which emits radiation in exactly the same spectral distribution as a blackbody at the same temperature, but with reduced intensity. The *total emissivity* (ϵ) of a body is the ratio of its total radiant emittance to that of a perfect blackbody at the same temperature. Emissivity is thus a measure of the radiation and absorption efficiency of a body. For a perfect blackbody $\epsilon = 1.0$, and most laboratory standard blackbodies are within a percent or two of this value. The table of Fig. 8.8 lists the total emissivity of a number of common materials. Note that emissivity varies with both wavelength and with temperature.

Radiation incident on a substance can be transmitted, reflected (or scattered), or absorbed. The transmitted, reflected, and absorbed fractions obviously must add up to 1.0. The absorbed fraction is the emissivity. Thus a material with either a high transmission or a high reflection must have a low emissivity.

When dealing with gray-bodies, it is necessary to insert the emissivity factor ϵ into the blackbody equations. Planck's law (Eq. 8.14), the Stefan-Boltzmann law (Eq. 8.15), and the Wien displacement law (Eq. 8.17) should be modified by multiplying the right-hand term by the appropriate value of ϵ . For many materials the emissiv-

Material	Total Emissivity	
Tungsten	500 K	0.05
	1000 K	0.11
	2000 K	0.26
	3000 K	0.33
	3500 K	0.35
Polished silver	650 K	0.03
Polished aluminum	300 K	0.03
Polished aluminum	1000 K	0.07
Polished copper	0.02–0.15	
Polished iron	0.2	
Polished brass	4–600 K	0.03
Oxidized iron	0.8	
Black oxidized copper	500 K	0.78
Aluminum oxide	80–500 K	0.75
Water	320 K	0.94
Ice	273 K	0.96–0.985
Paper	0.92	
Glass	293 K	0.94
Lampblack	273–373 K	0.95
Laboratory blackbody cavity	0.98–0.99	

Figure 8.8 The *total* emissivity of a number of materials.

ity is a function of wavelength. This is apparent from the fact that many substances (glass, for example) have a negligible absorption, and consequent low emissivity, at certain wavelengths, while they are almost totally absorbent at other wavelengths. In regions of the spectrum where this occurs, emissivity becomes spectral emissivity (ϵ_λ) and is treated just as any other spectral function. For many materials, emissivity will decrease as wavelength increases. It should also be noted that most materials show a variation of emissivity with temperature as well as wavelength, and precise work must take this into account. Emissivity usually increases with temperature.

Note that not all sources are continuous emitters. Gas discharge lamps at low pressure emit discrete spectral lines; the plot of spectral radiant emittance for such a source is a series of sharp spikes, although there is usually a low-level background continuum. In high-

pressure arcs, the spectral lines broaden and merge into a continuous background with less pronounced spikes.

Color temperature

Before leaving the subject of blackbody radiation, the concept of color temperature should be mentioned. The color temperature of a source of light is a colorimetric concept related to the apparent visual color of a source, not its temperature. For a blackbody, the color temperature is equal to the actual temperature in Kelvin. For other sources, the color temperature is the temperature of the blackbody which has the same apparent color as the source. Thus, exceedingly bright or dim sources may have the same color temperature, but radically different radiances or intensities. Color temperature usually runs about 150 K higher than filament temperature. Color temperature is extremely important in colorimetry and in color photography where fidelity of color rendition is important, but is little used in radiometry.

8.9 Photometry

Photometry deals with *luminous radiation*, that is, radiation which the human eye can detect. The basic photometric unit of radiant power is the lumen, which is defined as a luminous flux emitted into a solid angle of one steradian by a point source whose intensity is $\frac{1}{680}$ of that of 1 cm^2 of a blackbody at the solidification temperature of platinum (2042 K). From the preceding section, we know that a blackbody radiates energy throughout the entire electromagnetic spectrum. Chapter 5 indicated that the eye was sensitive to only a small interval of this spectrum and that its response to different wavelengths within this interval varied widely. Thus, if a source of radiation has a spectral power function $P(\lambda)$ ($\text{W } \mu\text{m}^{-1}$), the visual effect of this radiation is obtained by multiplying it by $V(\lambda)$,* the visual response function which is tabulated in Fig. 5.9. The effective visual power of a source is, therefore, the integral (or summation) of $P(\lambda) V(\lambda) d\lambda$ over the appropriate wavelength interval. From the definition of the lumen, it can be determined that one watt of radiant energy at the wavelength of maximum visual sensitivity ($0.555 \mu\text{m}$) is equal to 680 lumens. Therefore, the

*Note that $V(\lambda)$ is customarily the photopic (normal level of illumination and brightness) visual response curve. Under conditions of complete dark adaptation, the visual response for scotopic vision would be used. The conversion constant in Eq. 8.18 becomes 1746 instead of 680.

Source	Brightness, candles cm ⁻²
Sun (zenith) through atmosphere	1.6×10^5 cd/cm ²
Sun (zenith) above atmosphere	2.75×10^5
Sun (horizon)	6×10^2
Blue sky	0.8
Dark cloudy sky	4×10^{-3}
Night sky	5×10^{-9}
Moon	0.25
Exteriors—daylight (typical)	1
Exteriors—night (typical)	10^{-6}
Interiors—daylight (typical)	10^{-2}
Mercury arc—laboratory	10
Mercury arc—high pressure	5×10^5
Xenon arc	1.5×10^4 to 1.5×10^5
Carbon arc	10^4 to 10^5
Tungsten—3655 K (melting point)	5.7×10^3
3500 K	4.2×10^3
3000 K	1.3×10^3
Tungsten filament – ordinary lamp	5×10^2
– projection lamp	3×10^3
Blackbody—2040 K	60.0 (by definition)
—4000 K	2.5×10^4
—6500 K	3×10^5
Fluorescent lamp	0.6
Sodium lamp	6
Flame—candle, kerosene	1
Least perceptible brightness	5×10^{-11}
Least perceptible point source	2×10^{-8} cd @ 3 m distance
Star Sirius	1.5×10^6
Atom bomb	10^8
Lightning	8×10^6
Ruby laser	10^{14}
Metal halide lamp	4×10^4

Figure 8.9 Typical values for the brightness (luminance) of a number of sources.

luminous flux emitted by a source with a spectral power of $P(\lambda) \text{ W } \mu\text{m}^{-1}$ is given by

$$F = 680 \int V(\lambda) P(\lambda) d\lambda \text{ lumens} \quad (8.18)$$

The unit of luminous intensity is called the candle (or “candela”) and is so named because the original standard of intensity was an actual candle. A point source of one candlepower is one which emits one lumen into a solid angle of one steradian. A source of one candle intensity which radiates uniformly in all directions emits 4π lumens. From the definition of the lumen, it is apparent that a 1-cm^2 blackbody at 2042 K has an intensity of 60 candles.

Illumination, or illuminance, is the luminous flux per unit area incident on a surface. The most widely used unit of illumination is the foot-candle. One footcandle is one lumen incident per square foot. The misleading name footcandle resulted from the fact that it is the illumination produced on a surface one foot away from a source of one-candle intensity. The photometric term illuminance corresponds to irradiance in radiometry.

The term *brightness, or luminance*, corresponds to the term *radiance*. Brightness is the luminous flux emitted from a surface per unit solid angle per unit of area (projected on a plane normal to the line of sight). There are several commonly used units of brightness. The candle per square centimeter is equal to one lumen emitted per steradian per square centimeter. The lambert is equal to $1/\pi$ candles per square centimeter. The foot-lambert is equal to $1/\pi$ candles per square foot. The foot-lambert is a convenient unit for illuminating engineering work, since it is the brightness which results from one footcandle of illumination falling on a “perfect” diffusing surface. (Since one lumen is incident on the 1-ft^2 area under an illumination of one footcandle, the total flux radiated into a hemisphere of 2π ster. from a perfectly diffuse (lambertian) surface is just one lumen. As pointed out in Sec. 8.4 and Example A, the resulting brightness is $1/\pi$ lumen ster $^{-1}$ ft $^{-2}$, not $1/2\pi$ lumen ster $^{-1}$ ft $^{-2}$). The brightness of a number of sources is tabulated in Fig. 8.9 and natural illumination and reflectance levels are tabulated in Fig. 8.10.

The terminology of photometry has grown through engineering usage, and is thus far from orderly. Special terms have derived from special usages, and many such terms have survived. A tabulation of photometric units is given in Fig. 8.11.

Photometric calculations may be carried out exactly as radiometric calculations, using the relationships presented in Secs. 8.2 through 8.6. If lumens are substituted for watts in all the expressions, the computations are straightforward. When the starting and final data must be

Source	Illumination, footcandles
Direct sunlight	10,000 footcandles
Open shade	1,000
Overcast/dark day	10 to 100
Twilight	0.1 to 1.0
Full moon	0.01
Starlight	0.0001
Dark night	0.00001

(a)

Material	Reflectance
Asphalt	0.05
Trees, grass	0.20
Red brick	0.35
Concrete	0.40
Snow	0.85
Aluminum building	0.65
Glass window wall	0.70
Parking lot with cars	0.40

(b)

Figure 8.10 (a) Illumination levels produced by sources in nature. (b) Reflectance of a number of exteriors.

expressed in the special terminology of photometry (as opposed to what one might term the rational units of lumens, steradians, and square centimeters), then conversion factors may be necessary for each relationship. A very simple way of avoiding this difficulty is to convert the starting data to lumens, steradians, and square centimeters, complete the calculation, and then convert the results into the desired units.

For convenience, the basic relationships are repeated here in both radiometric (left column) and photometric (right column) form:

Radiant Intensity: $J = P/\Omega$

J is radiant intensity

P is the radiant power emitted into solid angle Ω

Luminous Intensity: $I = F/\Omega$

I is luminous intensity

F is the luminous flux emitted into solid angle Ω

Irradiance: $H = J/S^2 = J\Omega$

H is the irradiance incident on a surface a distance S from a point

Illumination (illuminance):

$$E = I/S^2 = I\Omega$$

E is the illumination incident on a surface a distance S from a point

Flux (Symbol F) lumen	defined in text
Intensity (Symbol I) candle ("candela")	one lumen per steradian emitted from a point source. 1/60 of the intensity of one square centimeter of a blackbody at 2042 K.
carcel	9.6 candles
hefner	0.9 candles
"old candle"	1.02 candles (candela)
Illumination (Symbol E) (Also called illuminance)	
footcandle	one lumen per square foot incident on a surface
phot	one lumen per square centimeter
lux	one lumen per square meter
meter-candle	one lumen per square meter
Brightness (Symbol B) (also called luminance)	
candle per square centimeter	one lumen emitted per steradian per square centimeter area projected normal to direction.
stilb	one candle per square centimeter
lambert	$1/\pi$ candles per square centimeter
foot-lambert	$1/\pi$ candles per square foot

Figure 8.11 Photometric quantities.

source of intensity J . Ω is the solid angle subtended by a unit area of the surface from the source.

$$H = \pi N \sin^2 \theta$$

H is the irradiance produced by a diffuse circular source of radiance N at a point from which the source diameter subtends 2θ .

$$H = N\omega$$

H is the irradiance produced by a diffuse source of radiance N at a point from which the area of the source subtends the solid angle ω .

$$H = T\pi N \sin^2 \theta$$

$$(H = TN\omega)$$

H is the irradiance at an image formed by an optical system of

source of intensity I . Ω is the solid angle subtended by a unit area of the surface from the source.

$$E = \pi B \sin^2 \theta$$

E is the illumination produced by a diffuse circular source of brightness (luminance) B at a point from which the source diameter subtends 2θ .

$$E = B\omega$$

E is the illumination produced by a diffuse source of brightness B at a point from which the area of the source subtends the solid angle ω .

$$E = T\pi B \sin^2 \theta = T\pi B/4(f/\#)^2 (m + 1)^2$$

$$(E = TB\omega) \quad \left[m = \left(\frac{s'}{f} - 1 \right) \right]$$

E is the illumination at an image formed by an optical system of trans-

transmission T whose exit pupil diameter (area) subtends an angle 2θ (solid angle ω) from the image point when object radiance is N .

Radiance: $N = P/(\pi A)$

N is the radiance of a diffuse source of area A which emits radiant power P into a hemisphere of 2π steradians.

mission T whose exit pupil diameter (area) subtends an angle 2θ (solid angle ω) from the image point when the object brightness is B .

Brightness (luminance):

$$B = F/(\pi A)$$

B is the brightness of a diffuse source of area A which emits luminous flux F into a hemisphere of 2π steradians.

Example C

It may be instructive to repeat Example A in photometric terms and to indicate at each step in the calculation the conversions to the various photometric units. We will use Fig. 8.5 again; the only change in the starting data will be that the source A will be assumed to have a brightness of $10 \text{ lumens ster}^{-1} \text{ cm}^{-2}$.

From Fig. 8.11, we note that the source brightness may also be expressed as $10 \text{ candles cm}^{-2}$, as 10 stilb , as $10\pi \text{ lamberts}$, or as $9290\pi \text{ foot-lamberts}$.

The illumination produced at point B is calculated from Eq. 8.7 (after rewriting it in photometric symbols)

$$\begin{aligned} H &= \pi N \sin^2 \theta \\ E &= \pi B \sin^2 \theta \\ &= \pi (10L \text{ ster}^{-1} \text{ cm}^{-2}) \left(\frac{1}{2} \right)^2 \\ &= 7.85 \text{ lumen cm}^{-2} \end{aligned}$$

Applying the cosine-fourth law, we find the illumination at C

$$\begin{aligned} E_C &= E_B \cos^4 45^\circ \\ &= 7.85 \times (0.707)^4 \\ &= 1.96 \text{ lumen cm}^{-2} \end{aligned}$$

Since there are 929 cm^2 per square foot

$$\begin{aligned} E_C &= 929 \times 1.96 = 1821 \text{ lumens ft}^{-2} \\ &= 1821 \text{ footcandles} \end{aligned}$$

Since the surface BC has a diffuse reflectivity of 70 percent, we can multiply the illumination in footcandles by 0.7 to obtain the brightness in foot-lamberts

$$B = 0.7 \times 1821 = 1275 \text{ foot-lamberts}$$

Similarly 0.7 times the illumination in lumens cm^{-2} will yield the brightness in lamberts

$$B = 0.7 \times 1.96 = 1.37 \text{ lamberts}$$

Or we can retain the lumen units, and determine that, with 1.96 lumen cm^{-2} falling on a surface 70 percent reflectivity, 1.37 lumen cm^{-2} will be emitted into a hemisphere, and, following our previous reasoning, compute the brightness as

$$\begin{aligned} B &= \frac{1.37}{\pi} \\ &= 0.44 \text{ lumen ster}^{-1} \text{ cm}^{-2} \\ &= 0.44 \text{ candle cm}^{-2} \end{aligned}$$

The illumination at E is determined from Eq. 8.11 as before

$$\begin{aligned} H &= TN\pi \sin^2 \theta \\ &= TN\omega \\ E &= TB\omega \\ &= 0.8 \times 0.44 \times 10^{-4} \\ &= 0.35 \times 10^{-4} \text{ lumen cm}^{-2} \\ &= 929 \times 0.35 \times 10^{-4} = 0.032 \text{ footcandles} \end{aligned}$$

8.10 Illumination Devices

Searchlight

A *searchlight* is one of the simpler, and at the same time one of the least understood, illuminating devices. It consists of a source of light (usually small) placed at the focal point of a lens or reflector. The image of the source is thus located at infinity. A common misconception is that the beam of light produced is a “collimated parallel bundle” which extends out to infinity with a constant diameter and a constant power density. A little consideration of the matter will

reveal the fallacy: the rays from any *point* on the source do indeed form a collimated parallel bundle, etc. However, a geometrical point on any source of finite brightness must emit zero energy, since a point has zero area, and therefore the “collimated bundle” of rays has zero energy.

With reference to Fig. 8.12, which shows a source S at the focal point of lens L , the image (S') will be located at infinity. Since source S subtends an angle α from lens L , the image S' will also subtend α . Now the illumination at a point on the axis will be determined by the brightness of the image and the solid angle subtended by the image. Thus, for points *near the lens*, the illumination is given by

$$E = TB\omega \quad (8.19)$$

which the reader will recognize as Eq. 8.8 rewritten in photometric symbols and with a transmission constant (T) added. B is the brightness of source S (since the brightness of an image equals the brightness of the object) and ω is the solid angle subtended by the image. (We have tacitly assumed ω to be small.) Now for a point at the lens, it is obvious that the solid angle ω subtended by the image S' is exactly equal to the solid angle subtended by the source S from the lens. Since S' is at infinity, this angle will not change as we shift our reference point a short distance along the axis away from the lens, and the illumination will remain constant in this region. However, at a distance $D = (\text{lens diameter})/\alpha$, the source image will subtend the same angle as the diameter of the lens, and for points more distant than D , the size of the solid angle subtended by the source of illumination will be limited by the lens diameter. This solid angle will obviously be equal to $(\text{area of lens})/d^2$ and the illumination beyond distance D will fall off with the square of the distance (d) to the lens. Thus, the equations governing the illumination produced by a searchlight are

$$D = \frac{\text{lens diameter}}{\alpha} \quad (8.20)$$

$$\text{for } d \leq D: E = TB\omega = (\text{a constant}) \quad (8.21)$$

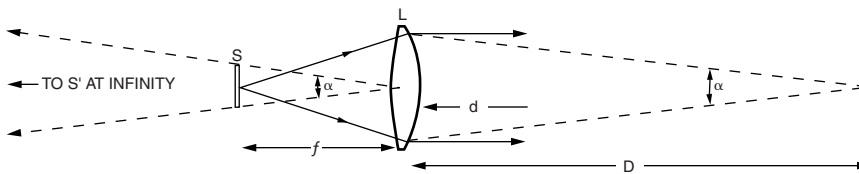


Figure 8.12 The optics of a searchlight.

$$\text{for } d \geq D: E = \frac{TB (\text{lens area})}{d^2} \quad (8.22)$$

The general technique used here is applicable to almost any illumination problem, and we can restate it in general terms as follows:

To determine the illumination at a point, the size and position of the source image, as seen from the point, are calculated. The pupils and windows of the system (again, as seen from the point) are determined. Then the illumination at the point is the product of the system transmission, the source brightness and the solid angle subtended by that area of the source which can be seen from the point through the pupils and windows of the system, multiplied by the cosine of the angle of incidence.

Note that for points (which lie within the beam) beyond the critical distance D , the searchlight acts as if it were a source of a diameter equal to that of the searchlight lens and a brightness TB . As mentioned in Sec. 8.6, this concept is quite useful in evaluating the illumination at an image point; here we find that it occasionally can be applied to points which are not image points.

The *beam candle power* of a searchlight is simply the intensity of the (point) source which would produce the same illumination at a great distance. A point source with an intensity of I candles will emit I lumens per steradian. A one-square-foot area placed d feet from the point source will subtend $1/d^2$ steradians from the source, and will thus be illuminated by I/d^2 lumens per square foot (footcandles). We can determine the necessary candle power for I by equating this illumination to that produced by the searchlight according to Eq. 8.22.

$$E = \frac{I}{d^2} = \frac{TB (\text{lens area})}{d^2} \quad (8.23)$$

and beam candlepower:

$$I = TB (\text{lens area})$$

where I is the beam candle power in lumens per steradian (or candles). Note that the lens area should be specified in the same units as the source brightness.

Projection condenser

The second illumination device we shall consider is the *projection condenser*, which is schematically diagrammed in Fig. 8.13. The purpose of the projector is to produce a bright and evenly illuminated image of the film on the screen. This could be achieved by placing a sheet of dif-

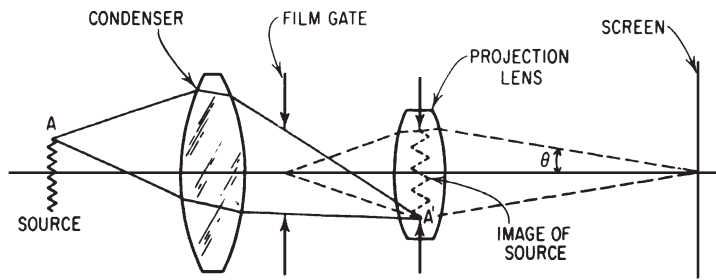


Figure 8.13 Schematic of a projection condenser system. The condenser forms an image of the source (lamp filament) in the aperture of the projection lens.

fusing material behind the film and illuminating this diffuser. The resultant image would be dim, because the maximum brightness which the image could achieve would be that of the diffuser, which would be considerably less than that of the lamp. The function of the condenser is to image the source in the pupil of the projection lens so that the lens aperture has the same brightness as the source. When this is done, the screen is illuminated according to Eq. 8.11, where the solid angle is that subtended by the source image (in the projection lens) from the screen. It is apparent that the maximum value for the screen illumination is limited by the size of the projection lens aperture. Therefore, the maximum screen illumination is achieved when the image of the source completely fills the aperture of the lens. This is required for all points within the field of view, and the condenser diameter must be sufficiently large so that it does not vignette, if maximum illumination at the edge of the picture is required. In this regard, note that the ray from the corner of the film to the opposite edge of the lens aperture is the most demanding. The cosine-fourth rule will, of course, reduce the illumination at points off the axis.

From the above, one might conclude that with a condenser of sufficient magnification, the image of a very small source could be magnified enough to fill the pupil of the projection lens. The necessary illuminating cone angle is determined by the film gate and its distance to the lens pupil (i.e., to the image of the source). In Chap. 2 we found that the magnification was given by $m = h'/h = u/u'$. The Abbe sine condition uses $m = \sin u/\sin u'$ for systems of reasonable image quality. Since u' in this case is fixed by the film gate, it is apparent that a large magnification will require a large value of u . The largest value that u can have is 90° with a sine of one; this establishes the limit on the magnification that can be attained. This limit can be expressed as

$$\left| \frac{P\alpha}{nS} \right| \leq 1.0 \quad (8.24)$$

where P is the aperture of the projection lens, α is the half-field angle of projection, n is the index in which the source is immersed (usually $n = 1.0$ for air), and S is the size of the source. It is impossible for Eq. 8.24 to exceed a value of 1.0; a value of 0.5 is typical of many systems. Note that a value of 0.5 corresponds to a working speed of $f/1.0$ and that a value of 1.0 would require a working speed of $f/0.5$. (Eq. 8.24 is analogous to Eq. 9.24 for detector systems.)

When the source is irregular in shape, as in “V” filament lamps for example, the solid angle for Eq. 8.11 is determined just as one might expect, by dividing the area of the actual image of the filament by the square of the distance to the screen. Condenser design is discussed in Sec. 13.4.

Telescope brightness

The apparent brightness of an image as seen by the eye is a function of the diameter of the pupil of the eye, since it determines the illumination of the retina, in accordance with Eq. 8.11a. When the eye is used with an optical instrument, such as a telescope, the exit pupil of the instrument enters the picture. If the exit pupil is larger than that of the eye, then the apparent brightness of the object seen through the instrument is equal to the brightness of the object (modified by transmission losses and index effects), since the solid angle subtended by the pupil from the retina is unchanged. When the instrument exit pupil is smaller than that of the eye, then the apparent brightness of the object is reduced in proportion to the relative areas of the pupils. The exception to these brightness relationships of object and image occurs when the object is smaller than the diffraction limit of the optical system (e.g., a star). Since this is not an extended source, all the energy in the retinal image is concentrated on a few retinal receptors, and when the magnification and aperture of a telescope are increased so that its exit pupil diameter stays the same, its effective collection area is increased (at the objective) so that more energy is concentrated on the same retinal cells (because the size of the retinal image is the same, being governed by the diffraction limit), resulting in an increase in the apparent brightness of the source. For example, if a high enough power telescope of large aperture is used, stars may be seen in daylight, since their apparent brightness is increased while that of the sky (as an extended object) is not.

Integrating sphere

An *integrating sphere* is often used in the measurement of light and light sources, and also as a uniform lambertian (diffuse) source of light. It is a hollow sphere, coated on the inside with a highly reflec-

tive white diffuse paint. If spot A on the inside of the sphere is illuminated, the light reflected from this spot produces an illumination at some other point B on the inside of the sphere. This illumination varies with the cosines of angles ϕ and θ made by the line connecting A and B with the normals to the sphere surface at A and B . Thus the illumination at B varies as

$$\frac{\cos \theta \cos \phi}{D^2} \quad (8.25)$$

where D is the distance from A to B , and this expression, for the inside of a sphere, is a constant. Thus the entire inner surface of the sphere is uniformly illuminated by the light reflected from the illuminated spot. If we cut two small holes in the sphere, one to admit light and the other (in a location not directly illuminated by the first hole) for a light sensor, we have a device which can read the amount of radiation admitted into the sphere without any variation of sensitivity resulting from the direction of the light, the size of the beam, or the position of the beam in the admitting hole. The total radiation emitted by a lamp or other source which is placed inside the sphere can readily be measured. Conversely, if the light sensor is replaced by a source of light, then the other hole becomes an almost perfect, uniform, unpolarized, lambertian source of radiation.

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Note: Titles preceded by an asterisk (*) are out of print.

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Exercises

1 A point source emits 10 W/ster toward a 4-in-diameter optical system. How much power is collected by the optical system when its distance from the source is (a) 10ft, (b) 1 mi?

ANSWER: (a) 8.73×10^{-3} W; (b) 3.13×10^{-8} W

2 A 10-candlepower point source illuminates a perfectly diffusing surface which is tilted at 45° to the line of sight to the source. What is the brightness of the surface if it is 10 ft from the source?

ANSWER: 0.0707 foot-lamberts, or 2.42×10^{-5} candles cm^{-2} .

3 A fluorescent lamp 10 in long and 1 in wide illuminates a slit, parallel to the lamp, which is 10 in long and 10 in from the lamp. If the lamp has a brightness of 0.5 candles cm^{-2} , what is the illumination (a) at the center of the slit, and (b) at the ends of the slit? (Hint: Divide the lamp into 10 one-inch-square sources.)

ANSWER: (a) 0.043 lumens cm^{-2} , or 40.2 footcandles

(b) 0.032 lumens cm^{-2} , or 29.9 footcandles

4 A 16-mm projector uses a 2-in $f/1.6$ projection lens and a lamp with a filament brightness of 3000 candles cm^{-2} . If the condenser fills the lens aperture with the filament image, what is the illumination produced on a screen 20 ft from the lens? Assume the transmission of the lens is 95 percent and the transmission of the condenser is 85 percent.

ANSWER: 47.9 footcandles, or 5.16×10^{-2} phot

5 (a) What is the spectral radiant emittance of a 1000-K blackbody in the region of 2 μm wavelength? What is the radiance? (b) If an idealized bandpass filter, transmitting only between 1.95 and 2.05 μm , is used, what is the total power falling on a 1- cm^2 detector placed 1 m from a 1- cm^2 1000-K blackbody? (Use Fig. 8.7.)

ANSWER: (a) emittance 0.89 $\text{W cm}^{-2} \mu\text{m}^{-1}$; radiance = $0.89/\pi = 0.283 \text{ W cm}^{-2} \text{ster}^{-1} \mu\text{m}^{-1}$

(b) $2.83 \times 10^{-6} \text{ W}$

6 Show that, for long projection distances, the maximum lumen output of a projector is given by

$$F = \frac{\pi ABT}{4 (f/\#)^2} \text{ lumens}$$

where A is the area of the film gate, B the source brightness, T the transmission of the system, and $(f/\#)$ is the relative aperture of the projection lens.