Chapter

6

Stops and Apertures

6.1 Introduction

In every optical system, there are apertures (or stops) which limit the passage of energy through the system. These apertures are the clear diameters of the lenses and diaphragms in the system. One of these apertures will determine the diameter of the cone of energy which the system will accept from an axial point on the object. This is termed the *aperture stop,* and its size determines the illumination (irradiance) at the image. Another stop may limit the size or angular extent of the object which the system will image. This is called the *field stop.* The importance of these stops to the photometry (radiometry) and performance of the system cannot be overemphasized.

The elements of an inexpensive camera system are sketched in Fig. 6.1 and illustrate both aperture and field stops in their most basic forms. The diaphragm in front of the lens limits the diameter of the bundle of rays that the system can accept and is thus the aperture stop. The mask adjacent to the film determines the angular field coverage of the system and is quite apparently the field stop of the camera.

Not all systems are as obvious as this, however, and we will now consider more complex arrangements. Because the theory of stops is readily explained by the use of a concrete example, the following discussions will be with reference to Fig. 6.2, which is a highly exaggerated sketch of a telescopic system focused on an object at a finite distance. The system shown consists of an objective lens, erector lens, eyelens, and two internal diaphragms. The objective forms an inverted image of the object. This image is then reimaged at the first focal

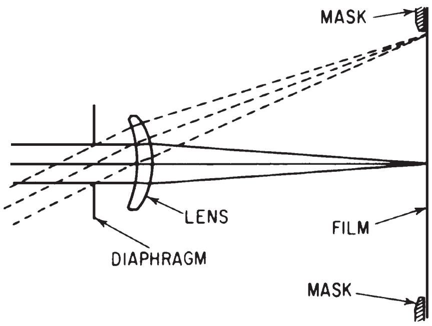

Figure 6.1 The elements of a simple box camera illustrate the functions of elementary aperture and field stops (the diaphragm and mask, respectively).

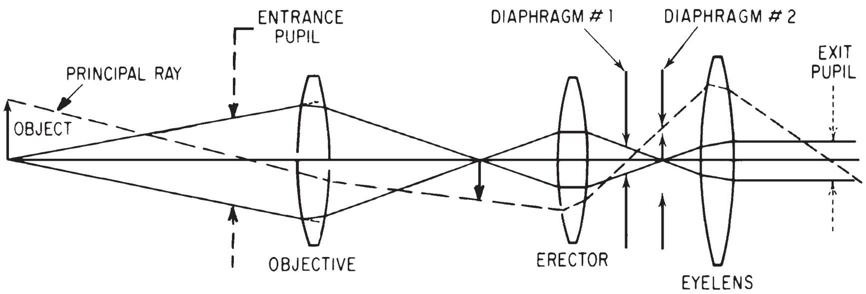

Figure 6.2 Schematic sketch of an optical system to illustrate the relationships between pupils, stops, and fields.

point of the eyelens by the erector lens, so that the eyelens forms the final image of the object at infinity.

6.2 The Aperture Stop and Pupils

By following the path of the axial rays (designated by solid lines) in Fig. 6.2, it can be seen that diaphragm #1 is the aperture of the system which limits the size of the axial cone of energy from the object. All of the other elements of the system are large enough to accept a bigger cone. Thus, diaphragm #1 is the aperture stop of the system.

The oblique ray through the center of the aperture stop is called the *principal,* or *chief, ray,* and is shown in the figure as a dashed line. The *entrance* and *exit pupils* of the system are the images of the aperture stop in object and image space, respectively. That is, the entrance pupil is the image of the aperture stop as it would be seen if viewed from the axial point on the object; the exit pupil is the aperture stop image as it would be seen if viewed from the final image plane (in this case, at an infinite distance). In the system of Fig. 6.2, the entrance pupil lies near the objective lens and the exit pupil lies to the right of the eyelens.

Notice that the initial and final intersections of the dashed principal ray with the axis locate the pupils, and that the diameter of the axial cone of rays at the pupils indicates the pupil diameters. It can be seen that, for any point on the object, the amount of radiation accepted by, and emitted from, the system is determined by the size and location of the pupils.

6.3 The Field Stop

By following the path of the principal ray in Fig. 6.2, it can be seen that another principal ray starting from a point in the object which is farther from the axis would be prevented from passing through the system by diaphragm #2. Thus, diaphragm #2 is the field stop of this system. The images of the field stop in object and image space are called the *entrance* and *exit windows,* respectively. In the system of Fig. 6.2, the entrance window is coincident with the object and the exit window is at infinity (which is coincident with the image). Note that the windows of a system do not coincide with the object and image unless the field stop lies in the plane of a real image formed by the system.

The angular field of view is determined by the size of the field stop, and is the angle which the entrance or exit window subtends from the entrance or exit pupil, respectively. The angular field in object space is frequently different from that in image space. (Alternate definition: the angular field of view is the angle subtended by the object or image from the first or second nodal point of the system, respectively. Thus, for nontelescopic systems in air, object and image field angles are equal according to this definition. Note that this definition cannot be applied to an afocal system, which has no nodal or principal points.)

6.4 Vignetting

The optical system of Fig. 6.2 was deliberately chosen as an ideal case in which the roles played by the various elements of the system are definite and clear-cut. This is not usually the situation in real optical systems, since the diaphragms and lens apertures often play dual roles.

Consider the system shown in Fig. 6.3, consisting of two positive lenses, *A* and *B*. For the axial bundle of rays, the situation is clear; the aperture stop is the clear aperture of lens *A,* the entrance pupil is at *A,* and the exit pupil is the image, formed by lens *B,* of the diameter of lens *A*.

Some distance off the axis, however, the situation is markedly different. The cone of energy accepted from point *D* is limited on its lower edge by the lower rim of lens *A* and on its upper edge by the upper rim of lens *B*. The size of the accepted cone of energy from point *D* is

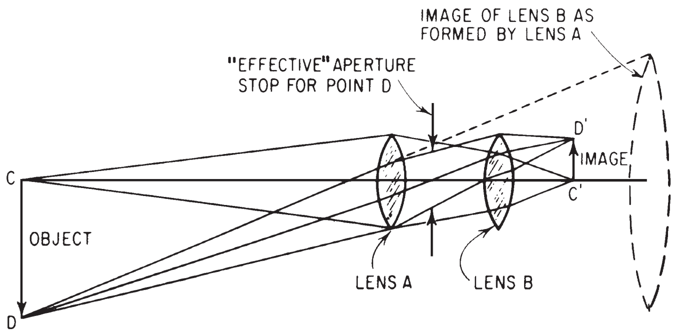

Figure 6.3 Vignetting in a system of separated components. The cone of rays from point *D* is limited by the lower rim of lens *A* and the upper rim of *B,* and is smaller than the cone accepted from point *C.* Note that the upper ray from *D* just passes through the image of lens *B* which is formed by lens *A.*

less than it would be if the diameter of lens *A* were the only limiting agency. This effect is called *vignetting,* and it causes a reduction in the illumination at the image point *D′*. It is apparent that for some object point still farther from the axis than point *D,* no energy at all would pass through the system; thus there is no field stop per se in this system as shown.

The appearance of the system when viewed from point *D* is shown in Fig. 6.4. The entrance pupil has become the common area of two circles, one the clear diameter of lens *A,* and the other the diameter of lens *B* as imaged by lens *A.* The dashed lines in Fig. 6.3 indicate the location and size of this image of *B,* and the arrows indicate the "effective" aperture stop which has a size, shape, and position completely different than that for the axial case.

In a photographic lens with an adjustable iris diaphragm, its location should be such that when stopped down to a small diameter, its clear aperture is centered in the vignetted oblique beam.

Example A

Let us determine the pupils, windows, and fields of an optical system of the type shown in Fig. 6.2, assuming the lenses to be "thin lenses." The elements of the system are as follows:

Objective	clear aperture = 2.3 in effective focal length = 10 in
Erector	clear aperture = 1.7 in effective focal length = 2 in
Eyelens:	clear aperture = 1.3 in effective focal length = 1 in

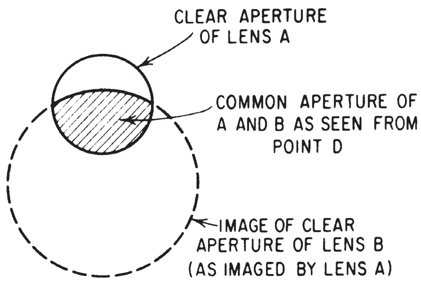

Figure 6.4 The apertures of the optical system of Fig. 6.3 as they are seen from point *D*.

Diaphragm #1: clear aperture = 0.25 in

Diaphragm #2: clear aperture = 0.7 in

Distance, object to objective: 50 in

Distance, objective to erector: 16.5 in

Distance, erector to eyelens: 5 in

Distance, erector to diaphragm #1: 2.38 in

Distance, erector to diaphragm #2: 4 in

We begin the analysis by tracing a paraxial ray from the object point on the axis, using the thin lens raytracing equations (2.41 and 2.42) of Chap. 2. We insert two zero-power elements in the system to represent the diaphragms, so that we can determine the ray heights at the diaphragms. We assume a nominal ray height of +1.0 at the objective lens, giving $u_1 = [1.0/(-50.) = +0.02$. The calculation is shown in the table of Fig. 6.5, lines 3 and 4.

To determine which element of the system limits the diameter of the axial cone of rays, we add to our tabulation lines 5 and 6, showing the clear aperture of each element (*CA*) and the ratio of the clear aperture to the height that the axial ray strikes the element (*CA/y*).The element for which this ratio is the smallest, in this case diaphragm #1, is the aperture stop. Because of the linear nature of the paraxial equations, we can get the *y* and *u* values for any other axial ray by multiplying each entry in lines 3 and 4 by the same constant. If we use for the constant the value of $^1/_2$ *CA/y* for diaphragm #1 (0.9645), we will get the data for a ray which just passes through the rim of diaphragm #1. This ray data is shown in lines 7 and 8 of the table. A comparison of the new *y* values of line 7 with the clear apertures of line 5 indicates that the ray will pass through all the other elements with room to spare.

To determine the locations of the pupils, we trace a ray through the center of the aperture stop (diaphragm #1) in each direction. The data of such a ray is shown in lines 9 and 10 of the table. We then determine the axial intersections of this ray in object and image space and find that the (apparent) entrance pupil is located 0.1631/0.02474 =

	Object plane		Objec-tive lens		Erector lens		Dia-phragm #1		Dia-phragm #2		Eyelens	
1. $\phi = 1/f$			+ 0.1		+ 0.5		0.0		0.0		+ 1.0	
2. d		+ 50.0		+ 16.5		+ 2.38		+ 1.62		1.0		
3. y	0.0		+ 1.0		− 0.32		− 0.1296		0.0		+ 0.8	
4. u		+ 0.02		− 0.08		+ 0.08		+ 0.08		+ 0.8		0.0
5. CA			+ 2.3		+ 1.7		+ 0.25		+ 0.7		+ 1.3	
6. CA/y			+ 2.3		+ 5.31		+ 1.929		Infinity		+ 16.25	
7. $y_o = 0.9645y$	0.0		+ 0.9645		− 0.3086		− 0.125		0.0		+ 0.07716	
8. $u_o = 0.9645u$		+ 0.01929		− 0.07716		+ 0.07716		+ 0.07716		+ 0.07716		0.0
9. y_p	+ 1.4		+ 0.1631		− 0.5142		0.0		+ 0.35		+ 0.5660	
10. u_p		− 0.02474		− 0.04105		+ 0.21605		+ 0.21605		+ 0.21605		− 0.35
11. $y_p + y_o$			+ 1.1276		− 0.8228		− 0.125		+ 0.35		+ 0.6432	
12. $y_p - y_o$			− 0.8013		− 0.2056		+ 0.125		+ 0.35		+ 0.4889	

Figure 6.5 Tabulation of the raytrace data for Example A.

+6.594 in to the right of the objective lens (note that this differs from Fig. 6.2) and that the exit pupil is 0.566/0.35 = +1.617 in to the right of the eyelens.

The diameter of the pupils is found from the ray data of lines 7 and 8 by determining the ray height in the plane of the pupils. Thus, the diameter of the entrance pupil is $2(0.9645 + 0.01929 \times 6.594)$, or 2.183 in, and the diameter of the exit pupil is $2(0.07716 - 0.0 \times 1.617)$, or 0.154 in.

A comparison of the values of CA/y_p would indicate that diaphragm #2 is the field stop. (The ray data in lines 9 and 10 of Fig. 6.5 have already been adjusted so that y_p at diaphragm #2 is equal to half of its clear aperture, in a manner analogous to that by which lines 7 and 8 were derived from lines 3 and 4.) The field of view is given by the slope of the principal ray which just skims through the field stop. This is the ray of lines 9 and 10; the object field is ±0.02474 radians and the image field is ±0.35 radians. The linear size of the object field is twice the height at which this ray strikes the object plane, or 2.8 in.

A check for vignetting could be made by tracing rays from an object point at the edge of the field through the upper and power rims of the entrance pupil. Again, because of the linearity of the paraxial equations, we can avoid this labor, since the height of the upper rim ray at an element is given by $y_p + y_0$ and that of the lower rim ray by $y_p - y_0$. (The values of y_0 and y_p are taken from the ray trace data which has been adjusted, i.e., lines 7 and 9.) This data is tabulated in lines 11 and 12 and a comparison with the clear apertures of the elements indicates that these rays pass through the system without vignetting.

An alternate technique for determining the aperture stop is to calculate the size and position of the image of *each* diameter of the system as seen from the object, i.e., as imaged by all the elements ahead of (or to the left of) the diameter. Then the diameter whose image subtends the smallest angle from the object is the aperture stop. A scale drawing of the images is handy when this technique is used.

6.5 Glare Stops, Cold Stops, and Baffles

A glare stop is essentially an auxiliary diaphragm located at an image of the aperture stop for the purpose of blocking out stray radiation. Depending on the system application, a glare stop may be called a *Lyot stop,* or in an infrared system, a *cold stop.* Figure 6.6 shows an erecting telescope in which the primary aperture stop is at the objective lens. Energy from sources outside the desired field of view, passing through the objective and reflecting from an internal wall, shield, or supporting member, can create a glare which reduces the contrast of the image formed by the system. In a long wavelength infrared

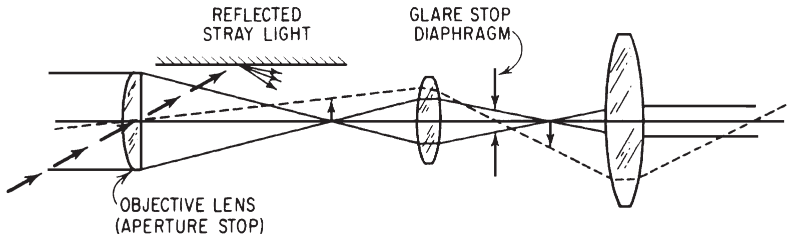

Figure 6.6 Stray light reflected from an inside wall of the telescope, is intercepted by the glare stop, which is located at the internal image of the objective lens.

system, the housing itself may be a source of unwanted thermal radiation. This radiation can be blocked out by an internal diaphragm which is an accurate image of the objective aperture. This stop is usually cooled and is located inside the evacuated detector Dewar. Since the stray radiation will appear to be coming from the wall, and thus from outside the objective aperture, it will be imaged on the opaque portion of the diaphragm. Another glare stop could conceivably be located at the exit pupil of this particular system, since it is real and accessible; however, it would make visual use of the instrument quite inconvenient.

In most systems the aperture stop is located at or very near the objective lens. This location gives the smallest possible diameter for the objective, and since the objective is usually the most expensive component (per inch of diameter), minimizing its diameter makes good economic sense. In addition, there are often aberration considerations which make this a desirable location. However, there are some systems, such as scanners, where the need to minimize the size and weight of the scanner mirror makes it necessary to put the stop or pupil at the scanner mirror rather than at the objective. This causes the objective to be larger, more costly, and more difficult to design.

In an analogous manner, field stops could be placed at both internal images to further reduce stray radiation. The principle here is straightforward. Once the primary field and aperture stops of a system are determined, auxiliary stops may be located at images of the primary stops to cut out glare. If the glare stops are accurately located and are the same size as the images of the primary stops (or slightly larger), they do not reduce the field or illumination, nor do they introduce vignetting.

Baffles are often used to reduce the amount of radiation that is reflected from walls, etc., in a system. Figure 6.7 shows a simple radiometer consisting of a collector lens and a detector in a housing. Assume that radiation from a powerful source (such as the sun) outside the field of view reflects from the inner walls of the mount onto the detector and obscures the measurement of radiation from the desired target, as shown in the upper half of the sketch. Under these conditions, there is no possibility of using an internal glare stop (since there is no internal image of the entrance pupil) and the internal walls of the mount must be baffled as shown in the lower half of the sketch (although an eternal hood or sunshade could also be used if circumstances permit).

The key to the efficient use of baffles is to arrange them so that no part of the detector can "see" a surface which is *directly* illuminated. The method of laying out a set of baffles is illustrated in Fig. 6.8. The dotted lines from the rim of the lens to the edge of the detector indicate the necessary clearance space, into which the baffles cannot intrude without obstructing part of the radiation from the desired field

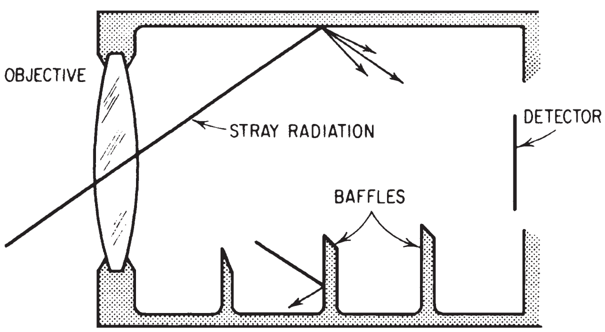

Figure 6.7 Stray (undesired) radiation from outside the useful field of this simple radiometer can be reflected from the inner walls of the housing and degrade the function of the system. Sharp-edged baffles, shown in the lower portion, trap this radiation and prevent the detector from "seeing" a directly illuminated surface.

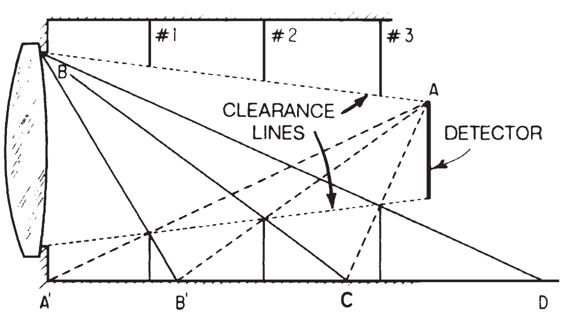

Figure 6.8 Construction for the systematic layout of baffles. Note that baffle #3 shields the wall back to point *D*; thus, all three baffles could be shifted forward somewhat, so that their coverages overlap.

of view. The dashed line *AA′* is a "line of sight" from the detector to the point on the wall where the extraneous radiation begins. The first baffle is erected to the intersection of *AA′* with the dotted clearance line. Solid line *BB′* indicates the path of stray light from the top of the lens to the wall. The area from Baffle #1 to *B′* is thus shadowed and "safe" for the detector to "see." The dashed line from *B′* to *A* is thus the safe line of sight, and baffle #2 at the intersection of *AB′* and the clearance line will prevent the detector from "seeing" the illuminated wall beyond *B′*. This procedure is repeated until the entire side wall is protected. Note that the inside edges of the baffles should be sharp and their surfaces rough and blackened.

The cast and machined baffles shown in Fig. 6.7 are obviously expensive to fabricate. Less expensive alternatives include washers constrained between spacers, or stamped, cup-shaped washers which can be cemented or press-fitted into place. This type of baffling is not necessary in all cases. Frequently, internal scattering can be sufficiently reduced by scoring or threading the offending internal surfaces of the mount. In this way, the reflections are broken up and scattered, reducing the amount of reflection and destroying any glare images. The use of a flat black paint is also highly advisable, although care must be taken to be sure that the paint remains both matte and black at near-grazing angles of incidence and at the application wavelength. Sandblasting to roughen the surface and blackening (for aluminum, black anodizing works well) is a simple and usually effective treatment. Another treatment is the application of black "flocked" paper. This can be procured in rolls, cut to size, and cemented to the offending surfaces; this is especially useful for large internal surfaces and for laboratory equipment.

Specialized flat black paints are available for specific applications and wavelengths. In the absence of special paints, Floquil brand flat black model locomotive paint usually can be found at the local hobby shop and makes a pretty good general-purpose flat black. A specialized anodizing process, Martin Optical Black (or Martin Infrablack for the infrared) is extremely effective (<0.2 percent reflective) but is very fragile.

6.6 The Telecentric Stop

A telecentric system is one in which the entrance pupil and/or the exit pupil is located at infinity. A telecentric stop is an aperture stop which is located at a focal point of an optical system. It is widely utilized in optical systems designed for metrology (e.g., comparators and contour projectors and in microlithography) because it tends to reduce the measurement or position error caused by a slight defocusing of the system. Figure 6.9a shows a schematic telecentric system. Note that the dashed principal ray is parallel to the axis to the left of the lens. If this system is used to project an image of a scale (or some other object), it can be seen that a small defocusing displacement of the scale does not change the height on the scale at which the principal ray strikes, although it will, of course, blur the image. Contrast this with Fig. 6.9b where the stop is at the lens, and the defocusing causes a proportional error in the ray height. The telecentric stop is also used where it is desired to project the image of an object with depth (along the axis), since it yields less confusing images of the edges of such an object.

6.7 Apertures and Image Illumination—*f*-Number and Cosine-Fourth

f-Number

When a lens forms the image of an extended object, the amount of energy collected from a small area of the object is directly proportional to the area of the clear aperture, or entrance pupil, of the lens. At the image, the illumination (power per unit area) is inversely proportional to the image area over which this object is spread. Now the aperture area is proportional to the square of the pupil diameter, and the image area is proportional to the square of the image distance, or focal

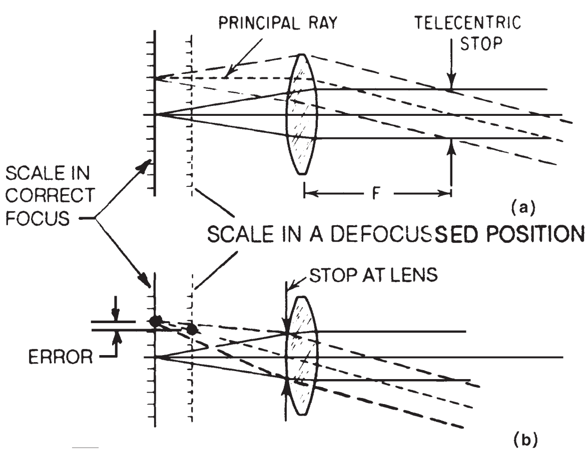

Figure 6.9 The telecentric stop is located at the focal point of the projection system shown, so that the principal ray is parallel to the axis at the object. When the object is slightly out of focus (dotted) there is no error in the size of the projected image as there is in the system with the stop at the lens, shown in the lower sketch.

length. Thus, the square of the ratio of these two dimensions is a measure of the relative illumination produced in the image.

The ratio of the focal length to the clear aperture of a lens system is called the relative aperture, *f*-number, or "speed" of the system, and (other factors being equal), the illumination in an image is inversely proportional to the square of this ratio. The relative aperture is given by:

$$f\text{-number} = \text{efl/clear aperture} \tag{6.1}$$

As an example, an 8-in focal length lens with a 1-in clear aperture has an *f*-number of 8; this is customarily written *f*/8 or *f*:8.

Another way of expressing this relationship is by the *numerical aperture* (usually abbreviated as N.A. or NA), which is the index of refraction (of the medium in which the image lies) times the sine of the half angle of the cone of illumination.

$$\text{Numerical aperture} = NA = n' \sin U' \tag{6.2}$$

Numerical aperture and *f*-number are obviously two methods of defining the same characteristic of a system. Numerical aperture is more conveniently used for systems that work at finite conjugates (such as microscope objectives), and the *f*-number is appropriately applied to systems for use with distant objects (such as camera lenses and telescope objectives). For aplanatic systems (i.e., systems corrected for coma and spherical aberration) with infinite object distances, the two quantities are related by:

$$f\text{-number} = \frac{1}{2\text{NA}} \tag{6.3}$$

The terms "fast" and "slow" are often applied to the *f*-number of an optical system to describe its "speed." A lens with a large aperture (and thus a small *f*-number) is said to be "fast," or to have a high "speed." A smaller aperture lens is described as "slow." This terminology derives from photographic usage, where a larger aperture allows a shorter (or faster) exposure time to get the same quantity of energy on the film and may allow a rapidly moving object to be photographed without blurring.

It should be apparent that a system working at finite conjugates will have an object-side numerical aperture as well as an image-side numerical aperture and that the ratio NA/NA′ = (object-side NA)/(image-side NA) must equal the absolute value of the magnification. The term "working *f*-number" is sometimes used to describe the numerical aperture in *f*-number terms. If we use the terms "infinity *f*-number" for the *f*-number defined in Eq. 6.1, then the image-side working f-number is equal to the infinity f-number times $(1 - m)$, where m is the magnification.

Another term that is occasionally encountered is the *T-stop,* or T-number. This is analogous to the f-number, except that it takes into account the transmission of the lens. Since an uncoated, many-element lens made of exotic glass may transmit only a fraction of the light that a low-reflection coated lens of simpler construction will transmit, such a speed rating is of considerable value to the photographer. The relationship between f-number, T-number, and transmission is

$$T\text{-number} = \frac{f\text{-number}}{\sqrt{\text{transmission}}} \tag{6.4}$$

Cosine-to-the-fourth

For off-axis image points, even when there is no vignetting, the illumination is usually lower than for the image point on the axis. Figure 6.10 is a schematic drawing showing the relationship between exit pupil and image plane for point A on axis and point H off axis. The illumination at an image point is proportional to the solid angle which the exit pupil subtends from the point.

The solid angle subtended by the pupil from point A is the area of the exit pupil divided by the square of the distance OA. From point $H,$ the solid angle is the projected area of the pupil divided by the square of the distance OH. Since OH is greater than OA by a factor equal to $1/\cos\theta$, this increased distance reduces the illumination by a factor of $\cos^2\theta$. The exit pupil is viewed obliquely from point $H,$ and its projected area is reduced by a factor which is *approximately* $\cos\theta$. (This is a fair approximation if OH is large compared to the size of the pupil; for high-speed lenses used at large obliquities, it may be subject to significant errors. See Example A in Chap. 8 for an exact expression.)

Thus the illumination at point H is reduced by a factor of $\cos^3\theta$. This is, however, true for illumination on a plane normal to the line OH

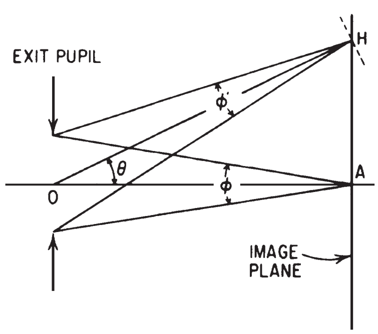

Figure 6.10 Relationship between exit pupil and image points, used to demonstrate that the illumination at H is $\cos^4\theta$ times that at A.

(indicated by the dashed line in Fig. 6.10). We want the illumination in the plane *AH*. An illumination of *x* lumens per square foot on the dashed plane will be reduced on plane *AH* because the same number of lumens is spread over a greater area in plane *AH*. The reduction factor is cos θ, and combining all the factors we find that

$$\text{Illumination at } H = \cos^4 \theta \text{ (illumination at } A) \tag{6.5}$$

The importance of this effect on wide-angle lenses can be judged from the fact that $\cos^4 30° = 0.56$, $\cos^4 45° = 0.25$, and $\cos^4 60° = 0.06$. It can be seen that the illumination on the film in a wide-angle camera will fall off quite rapidly.

Note that the preceding has been based on the assumption that the pupil diameter is constant (with respect to θ) and that θ is the angle formed in image space (although many people apply it to the field angle in object space). The "cosine fourth law" can be modified if the construction of the lens is such that the apparent size of the pupil increases for off-axis points, or if a sufficiently large amount of barrel distortion is introduced to hold θ to smaller values than one would expect from the corresponding field angle in object space. Certain extreme wide-angle camera lenses make use of these principles to increase off-axis illumination. The $\cos^4$ effect is in addition to any illumination reduction caused by vignetting. It should be remembered that the cosine-fourth effect is *not* a "law" but a collection of four cosine factors which may or may not be present in a given situation.

6.8 Depth of Focus

The concept of depth of focus rests on the assumption that for a given optical system, there exists a blur (due to defocusing) of small enough size such that it will not adversely affect the performance of the system. The *depth of focus* is the amount by which the image may be shifted longitudinally with respect to some reference plane (e.g., film, reticle) and which will introduce no more than the acceptable blur. The *depth of field* is the amount by which the object may be shifted before the acceptable blur is produced. The size of the acceptable blur may be specified as the linear diameter of the blur spot (as is common in photographic applications) (Fig. 6.11) or as an angular blur, i.e., the angular subtense of the blur spot from the lens. Thus, the linear and angular blurs (B and β, respectively and the distance D are related by

$$\beta = \frac{B}{D} = \frac{B'}{D'} \tag{6.6}$$

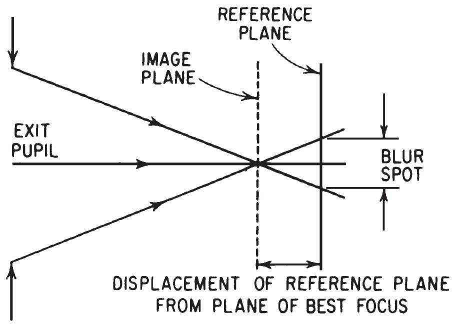

Figure 6.11 When an optical system is defocused, the image of a point becomes a blurred spot. The size of the blur is determined by the relative aperture of the system and the focus shift.

for a system in air, where the primed symbols refer to the image-side quantities.

Angular depth of focus

From Fig. 6.12, it can be seen that the depth of field δ for a system with a clear aperture A can be obtained from the relationship

$$\frac{\delta}{\beta\,(D \pm \delta)} = \frac{D}{A}$$

This expression can be solved for the depth of field, giving

$$\delta = \frac{D^2\beta}{(A \pm D\beta)} = \frac{DB}{(A \pm B)} \tag{6.7}$$

Note that the depth of field *toward* the optical system is smaller than that *away* from the system. When δ is small in comparison with the distance D, this reduces to

$$\delta = \frac{D^2\beta}{A} = \frac{D\beta}{A} \tag{6.8}$$

For the image side, the relationship is

$$\delta' = \frac{D'^2\beta}{A} = \frac{F^2\beta}{A} = F\beta\,(f/\#) = B'(f/\#) \tag{6.9}$$

where the second, third, and fourth forms of the right-hand side apply when the image is at the focal point of the system, and F is the system focal length.

The depth of focus in terms of linear blur-spot size B can be obtained by substituting Eq. 6.6 into the above. Also, note that the depth of field δ and the depth of focus δ′ are related by the longitudinal magnification of the system, so that

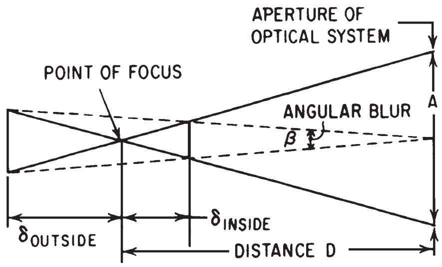

Figure 6.12 Relationships used to determine the longitudinal depth of field in terms of a tolerable angular blur.

$$\delta' = \overline{m} \approx m^2\delta \tag{6.10}$$

The *hyperfocal distance* of a system is the distance at which the system must be focused so that the depth of field extends to infinity. If ($D + \delta$) equals infinity, then ß is equal to A/D, so that

$$D\ (\text{hyperfocal}) = \frac{A}{\beta} = \frac{F\Delta}{B} \tag{6.11}$$

The photographic depth of focus

The photographic depth of focus is based on the concept that a defocus blur which is smaller than a silver grain in the film emulsion will not be noticeable. This concept also can be applied to pixel size in, for example, a charge-coupled device (CCD). If the acceptable blur diameter is B, then the depth of focus (at the image) is simply

$$\delta' = \pm B(f\text{-number})$$
$$\delta' = \pm \frac{B}{2\text{NA}} \tag{6.12}$$

The corresponding depth of field (at the object) is from D_{near} to D_{far}, where

$$D_{\text{near}} = \frac{fD\,(A + B)}{(fA - DB)} \tag{6.13}$$

$$D_{\text{far}} = \frac{fD\,(A - B)}{(fA + DB)} \tag{6.14}$$

and the hyperfocal distance is simply

$$D_{\text{hyp}} = \frac{-fA}{B} \tag{6.15}$$

where D = the nominal distance at which the system is focused (note that, by our sign convention, D is normally negative)

A = the diameter of the entrance pupil of the lens
f = the focal length of the lens

Note that there are several false assumptions here. We assume that the image is a perfect point, with no diffraction effects. We also assume that the lens has no aberrations and that the blurring on both sides of the focus is the same. None of these assumptions is correct, but the equations above *do* give a usable model for the depth of focus. In practice, the acceptable blur diameter B is usually determined empirically by examining a series of defocused images to decide the level of acceptability; the equations above are then fitted to the results.

6.9 Diffraction Effects of Apertures

Even if we assume that an infinitely small point source of light is possible, no lens system can form a true point image, even though the lens be perfectly made and absolutely free of aberrations. This results from the fact that light does not really travel in straight-line rays, but behaves as a wave motion, bending around corners and obstructions to a small but finite degree.

According to Huygen's principle of light-wave propagation, each point on a wave front may be considered as a source of spherical wavelets; these wavelets reinforce or interfere with each other to form the new wave front. When the original wave front is infinite in extent, the new wave front is simply the envelope of the wavelets in the direction of propagation. At the other extreme, when the wave front is limited by an aperture to a very small size (say, to the order of a half wavelength), the new wave front becomes spherical about the aperture. Figure 6.13 shows a plane wavefront incident on a slit *AC,* which is in front of a perfect lens. The lens is focused on a screen, *EF.* We wish to determine the nature of the illumination on the screen. Since the lens of Fig. 6.13 is assumed perfect, the optical path lengths *AE, BE,* and *CE* are all equal and the waves will arrive in phase at *E*, reinforcing each other to produce a bright area. For Huygen's wavelets

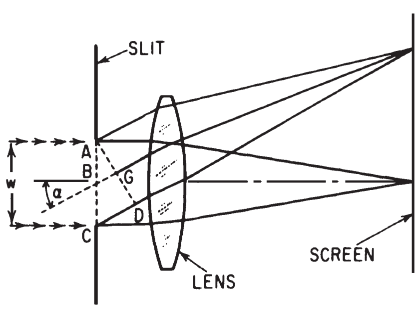

Figure 6.13

starting from the plane wave front in a direction indicated by angle α, the paths are different; path *AF* differs from path *CF* by the distance *CD*. If *CD* is an integral number of wavelengths, the wavelets from *A* and *C* will reinforce at point *F*. If *CD* is an odd number of half wavelengths, a cancellation will occur. The illumination at *F* will be the summation of the contributions from each incremental segment of the slit, taking the phase relationships into account. It can be readily demonstrated that when *CD* is an integral number of wavelengths, the illumination at *F* is zero, as follows: if *CD* is one wavelength, then *BG* is one-half wavelength and the wavelets from *A* and *B* cancel. Similarly, the wavelets from the points just below *A* and *B* cancel and so on down the width of the slit. If *CD* is *N* wavelengths, we divide the slit into 2*N* parts (instead of two parts) and apply the same reasoning. Thus, there is a dark zone at *F* when

$$\sin \alpha = \frac{\pm N\lambda}{w}$$

where N = any integer
λ = the wavelength of the light
w = the width of the slit

Thus, the illumination in the plane *EF* is a series of light and dark bands. The central bright band is the most intense, and the bands on either side are successively less intense. One can realize that the intensity should diminish by considering the situation when *CD* is 1.5λ, 2.5λ, etc. When *CD* is 1.5λ, the wavelets from two-thirds of the slit can be shown (as in the preceding paragraph) to interfere and cancel out, leaving the wavelets from one-third of the aperture; when *CD* is 2.5λ, only one-fifth of the slit is uncanceled. Since the "uncanceled" wavelets are neither exactly in nor exactly out of phase, the illumination at the corresponding points on the screen will be less than one-third or one-fifth of that in the central band.

For a more rigorous mathematical development of the subject, the reader is referred to the references following this chapter. The mathematical approach is one of integration over the aperture, combined with a suitable technique for the addition of the wavelets which are neither exactly in nor exactly out of phase. This approach can be applied to rectangular and circular apertures as well as to slits.

For a rectangular aperture, the illumination on the screen is given by

$$I = I_0 \frac{\sin^2 m_1}{m_1^2} \cdot \frac{\sin^2 m_2}{m_2^2} \tag{6.16}$$

$$m_i = \frac{\pi w_i \sin \alpha_i}{\lambda} \qquad i = 1,2 \tag{6.17}$$

In these expressions λ is the wavelength, w the width of the exit aperture, α the angle subtended by the point on the screen, m_1 and m_2 correspond to the two principal dimensions, w_1 and w_2, of the rectangular aperture and I_0 is the illumination at the center of the pattern.

When the aperture is circular, the illumination is given by

$$I = I_0 \left[1 - \frac{1}{2}\left(\frac{m}{2}\right)^2 + \frac{1}{3}\left(\frac{m^2}{2^2 2!}\right)^2 - \frac{1}{4}\left(\frac{m^3}{2^3 3!}\right)^2 + \frac{1}{5}\left(\frac{m^4}{2^4 4!}\right)^2 - \cdots \right]^2$$

$$= I_0 \left[\frac{2J_1(m)}{m} \right]^2 \tag{6.18}$$

where m is given by Eq. 6.17 with the obvious substitution of the diameter of the circular exit aperture for the width, w, and $J_1(\ )$ is the first-order Bessel function. The illumination pattern consists of a bright central spot of light surrounded by concentric rings of rapidly decreasing intensity. The bright central spot of this pattern is called the *Airy disk.*

We can convert from angle α to Z, the radial distance from the center of the pattern, by reference to Fig. 6.14. If the optical system is reasonably aberration-free, then

$$l' = \frac{-w}{2 \sin U'}$$

and to a close approximation, when α is small

$$Z = \frac{l'\alpha}{n'} = \frac{-\alpha w}{2n' \sin U'} \tag{6.19}$$

The table of Fig. 6.15 lists the characteristics of the diffraction patterns for circular and slit apertures. The table is derived from Eqs. 6.16 and 6.18, but the data is given in terms of Z and $\sin U'$ rather than α and w. Note that $n' \sin U'$ is the numerical aperture NA of the optical system.

Notice that 84 percent of the energy in the pattern is contained in the central spot, and that the illumination in the central spot is almost 60 times that in the first bright ring. Ordinarily the central spot and

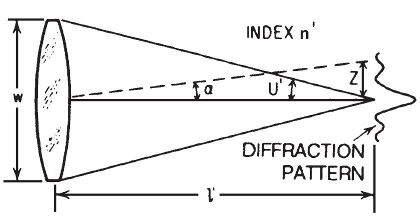

Figure 6.14

Ring (or band)	Circular Aperture Z	Peak Illumination	Energy in Ring	Slit Aperture Z	Peak Illumination
Central maximum	0	1.0	83.9%	0	1.0
1st dark ring	$0.61\ \lambda/n'\sin U'$	0.0		$0.5\ \lambda/n'\sin U'$	0.0
1st bright ring	$0.82\ \lambda/n'\sin U'$	0.017	7.1%	$0.72\ \lambda/n'\sin U'$	0.047
2d dark ring	$1.12\ \lambda/n'\sin U'$	0.0		$1.0\ \lambda/n'\sin U'$	0.0
2d bright ring	$1.33\ \lambda/n'\sin U'$	0.0041	2.8%	$1.23\ \lambda/n'\sin U'$	0.017
3rd dark ring	$1.62\ \lambda/n'\sin U'$	0.0		$1.5\ \lambda/n'\sin U'$	0.0
3rd bright ring	$1.85\ \lambda/n'\sin U'$	0.0016	1.5%	$1.74\ \lambda/n'\sin U'$	0.0083
4th dark ring	$2.12\ \lambda/n'\sin U'$	0.0		$2.0\ \lambda/n'\sin U'$	0.0
4th bright ring	$2.36\ \lambda/n'\sin U'$	0.00078	1.0%	$2.24\ \lambda/n'\sin U'$	0.0050
5th dark ring	$2.62\ \lambda/n'\sin U'$			$2.5\ \lambda/n'\sin U'$	0.0

Figure 6.15 Tabulation of the size of and distribution of energy in the diffraction pattern at the focus of a perfect lens.

the first two bright rings dominate the appearance of the pattern, the other rings being too faint to notice. The illumination in a diffraction pattern is plotted in Fig. 6.16. One should bear in mind the fact that these energy distributions apply to perfect, aberration-free systems with circular or slit apertures which are uniformly transmitting and which are illuminated by wave fronts of uniform amplitude. The presence of aberrations will, of course, modify the distribution as will any nonuniformity of transmission or wave-front amplitude (see, for example, Sec. 6.11).

6.10 Resolution of Optical Systems

The diffraction pattern resulting from the finite aperture of an optical system establishes a limit to the performance which we can expect from even the best optical device. Consider an optical system which images two equally bright point sources of light. Each point is imaged as an Airy disk with the encircling rings, and if the points are close, the diffraction patterns will overlap. When the separation is such that it is just possible to determine that there are two points and not one, the points are said to be resolved. Figure 6.17 indicates the summation of the two diffraction patterns for various amounts of separation. When the image points are closer than $0.5\lambda/\mathrm{NA}$ (NA is the numerical aperture of the system and equals $n'\sin U'$), the central maxima of both patterns blend into one and the combined patterns may appear to be due to a single source. At a separation of $0.5\lambda/\mathrm{NA}$ the duplicity of the image points is detectable, although there is no minimum between the maxima from the two patterns. This is *Sparrow's criterion* for resolution. When the image separation reaches $0.61\lambda/\mathrm{NA}$, the maximum

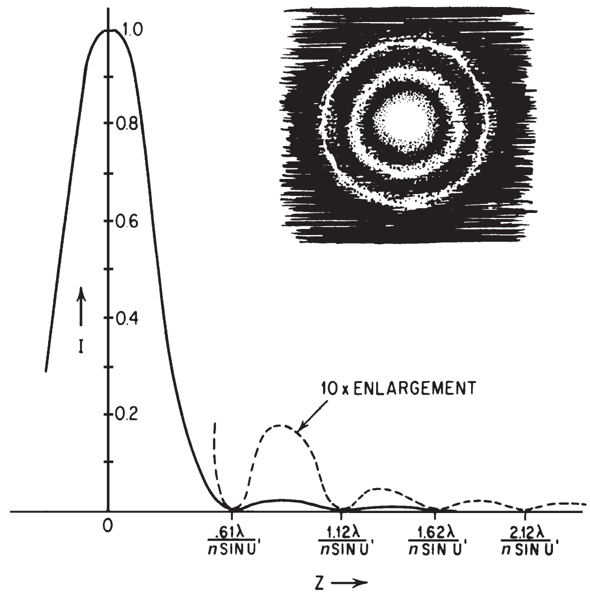

Figure 6.16 The distribution of illumination in the Airy disk. The appearance of the Airy disk is shown in the upper right.

of one pattern coincides with the first dark ring of the other and there is a clear indication of two separate maxima in the combined pattern. This is *Lord Rayleigh's criterion* for resolution and is the most widely used value for the limiting resolution of an optical system.*

From the tabulation of Fig. 6.15, we find that the distance from the center of the Airy disk to the first dark ring is given by

$$Z = \frac{0.61\lambda}{n' \sin U'} = \frac{0.61\lambda}{NA} = 1.22\lambda\ (f/\#) \qquad (6.20)$$

This is the separation of two image points corresponding to the Rayleigh criterion for resolution. This expression is widely used in determining the limiting resolution for microscopes and the like. For resolution at the image, the NA of the image cone is used; for resolution at the object, the NA of the object cone is used.

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*The diffraction pattern of two point images will always differ somewhat from the diffraction pattern of a single point. It is thus possible to detect the presence of two points (as opposed to one) even in cases where the two points cannot be visually resolved or separated. This is the source of the occasional claims that a system "exceeds the theoretical limit of resolution." In Chap. 11 it is shown that there is a true limit on the resolution of a sinusoidal *line* target; the limit on the spatial frequency is $v_0 = 2\text{NA}/\lambda = 1/\lambda(f/\#)$.

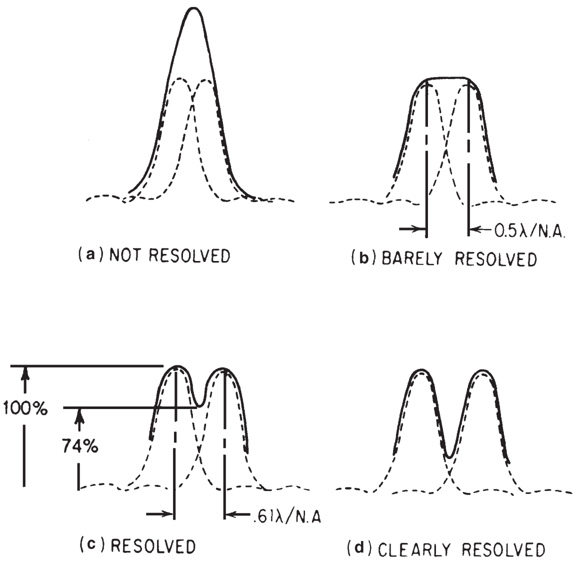

Figure 6.17 The dashed lines represent the diffraction patterns of two point images at various separations. The solid line indicates the combined diffraction pattern. Case (b) is the Sparrow criterion for resolution. Case (c) is the Rayleigh criterion.

To evaluate the performance limits of telescopes and other systems working at long object distances, an expression for the angular separation of the object points is more useful. Rearranging Eq. 6.19 and substituting the limiting value of Z from Eq. 6.20, we get, in radian measure,

$$\alpha = \frac{1.22\lambda}{w} \text{ radians} \tag{6.21}$$

For ordinary visual instruments, λ may be taken as 0.55 μm, and using $4.85 \cdot 10^{-6}$ radians for 1 second of arc, we find that

$$\alpha = \frac{5.5}{w} \text{ seconds of arc} \tag{6.22}$$

when w is the aperture diameter expressed in inches. By a series of careful observations, the astronomer Dawes found that two stars of equal brightness could be visually resolved when their separation was $4.6/w$ seconds. Notice that if the Sparrow criterion is used instead of the Rayleigh criterion in Eq. 6.22, the limiting resolution angle is $4.5/w$ seconds, which is in close agreement with Dawes' findings.

It is worth emphasizing here that the *angular* resolution limit is a direct function of wavelength and an inverse function of the aperture of the system. Thus, the limiting resolution is improved by reducing the wavelength or by increasing the aperture. Note that focal length or working distance do not directly affect the angular resolution. The *linear* resolution is governed by the wavelength and the numerical aperture (NA or *f*-number), and *not* by the aperture diameter.

In an instrument such as a spectroscope, where it is desired to separate one wavelength from another, the measure of resolution is the smallest wavelength difference, $d\lambda$, which can be resolved. This is usually expressed as $\lambda/d\lambda$; thus, a resolution of 10,000 would indicate that the smallest detectable difference in wavelength was 1/10,000 of the wavelength upon which the instrument was set.

For a prism spectroscope, the prism is frequently the limiting aperture, and it can be shown that when the prism is used at minimum deviation, the resolution is given by

$$\frac{\lambda}{d\lambda} = B\,\frac{dn}{d\lambda} \tag{6.23}$$

where B is the length of the base of the prism and $dn/d\lambda$ is the dispersion of the prism material.

A diffraction grating consists of a series of precisely ruled lines on a clear (or reflecting) base. Light can pass directly through a grating, but it is also diffracted. As with the slit aperture discussed above, at certain angles the diffracted wavelets reinforce, and maxima are produced when

$$\sin\alpha = \frac{m\lambda}{S} \pm \sin I \tag{6.24}$$

where λ is the wavelength, I is the angle of incidence, S is the spacing of the grating lines, m is an integer, called the *order* of the maxima, and the positive sign is used for a transmission grating, the negative for a reflecting. (Note that a sinusoidal grating has only a first order.) Since α depends on the wavelength λ, such a device can be used to separate the diffracted light into its component wavelengths. When used as indicated in Fig. 6.18, the resolution of a grating is given by

$$\frac{\lambda}{d\lambda} = mN \tag{6.25}$$

where m is the order and N is the total number of lines in the grating (assuming the size of the grating to be the limiting aperture of the system).

6.11 Diffraction of a Gaussian (Laser) Beam

The illumination distribution in the image of a point as described in Secs. 6.9 and 6.10 was based on the assumptions that the optical system was perfect and that both the transmission and the wave-front

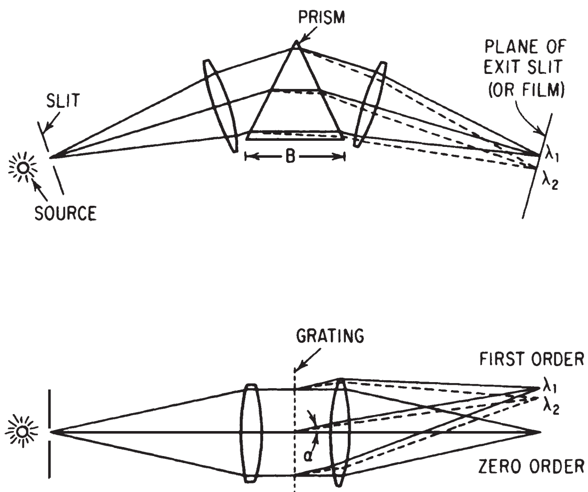

Figure 6.18 (Upper) Prism spectrometer. (Lower) Grating spectrometer.

amplitude were uniform over the aperture. Any change in the intensity distribution in the beam will change the diffraction pattern from that described above. Obviously, a similar change in the transmission of the aperture will produce the same effects.

A "gaussian beam" is one whose intensity cross section follows the equation of a gaussian, $y = e^{-x^2}$. Laser output beams closely approximate gaussian beams. From mathematics we know that exponential functions, such as the gaussian are extremely resistant to transformations (consider, for example, the integral or differential of e^{-x}). Similarly, a gaussian beam tends to remain a gaussian beam, as long as it is "handled" by reasonably aberration-free optics, and the diffraction image of a point also has a gaussian distribution of illumination.

The distribution of intensity in a gaussian beam is illustrated in Fig. 6.19 and can be described by Eq. 6.26.

$$I\,(r) = I_o e^{-2r^2/w^2} \tag{6.26}$$

where $I\,(r)$ = the beam intensity at a distance r from the beam axis
I_0 = the intensity on axis
r = the radial distance
e = 2.718. . . .
w = the radial distance at which the intensity falls to I_0/e^2, i.e., to 13.5 percent of its central value. This is usually referred to as the beam width, although it is a semidiameter. It encompasses 86.5 percent of the beam power.

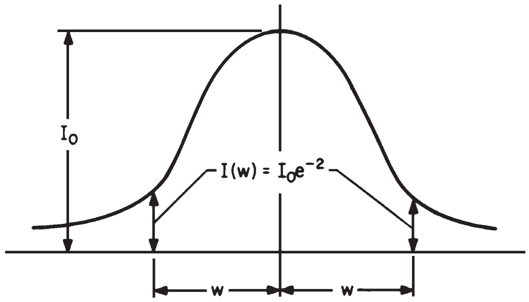

Figure 6.19 Gaussian beam intensity profile.

Beam power

By integration of Eq. 6.26 we find the total power in the beam to be given by

$$P_{tot} = {}^1/_2\, \pi I_0 w^2 \tag{6.27}$$

The power passed through a centered circular aperture of radius a is given by

$$P(a) = P_{tot}(1 - e^{-2a^2/w^2}) \tag{6.28}$$

The power passed by a centered slit of width $2s$ is given by

$$P(s) = P_{tot} \cdot \text{erf}\left(\frac{s\sqrt{2}}{w}\right) \tag{6.29}$$

where $\text{erf}(u) = \int_0^u e^{-t^2}\,dt$ = the error function, which is tabulated in mathematical handbooks.

Diffraction spreading of a gaussian beam

A gaussian beam has a narrowest width at some point, which is called the "waist." This point may be near where the beam is focused or near where it emerges from the laser. As the beam progresses, it spreads out according to the following equation:

$$w_z^2 = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right] \tag{6.30}$$

where w_z = the semidiameter of the beam (i.e., to the $1/e^2$ points) at a longitudinal distance z from the beam waist.

w_0 = the semidiameter of the beam (to the $1/e^2$ points) at the beam waist.

λ = the wavelength

z = the distance along the beam axis from the waist to the plane of w_z

At large distances it is convenient to know the angular beam spread. Dividing both sides of Eq. 6.30 by z^2, then, as z approaches infinity, we get

$$\frac{\alpha}{2} = \left.\frac{w_z}{z}\right|_{z\to\infty} = \frac{\lambda}{\pi w_0} = \frac{2\lambda}{\pi\,(2w_0)} \quad \text{or} \quad \alpha = \frac{4\lambda}{\pi\,(2w_0)} = \frac{1.27\lambda}{\text{diameter}} \tag{6.31}$$

where α is the angular beam spread in radians between the $1/e^2$ points. For many applications, the gaussian diffraction blur at the image plane can be found by simply multiplying α from Eq. 6.31 by the image conjugate distance (s' from Chap. 2).

Beam truncation

The effect of beam truncation, i.e., stopping down or cutting off the outer regions of the beam, is discussed by Campbell and DeShazer. They show that if the diameter of the beam is not reduced below $2(2w)$, where w is the beam semidiameter at the $1/e^2$ points, then the beam intensity distribution remains within a few percent of a true gaussian distribution. If the clear aperture is reduced below this value, it will introduce structure (i.e., rings) into the irradiance patterns, and the pattern gradually approaches Eq. 6.18 as the aperture is reduced.

A lens aperture large enough to pass a beam with a diameter of $4w$ is obviously very inefficient from a radiation transfer standpoint. For this reason, most systems truncate the beam, very often to the $1/e^2$ diameter, and the diffraction pattern is altered accordingly. If the beam is truncated down to 61 percent of the $1/e^2$ diameter, it is difficult to see the difference from a uniform beam.

Size and location of a new waist formed by a perfect optical system

When a gaussian beam passes through an optical system, a new waist is formed. Its size and location are determined by diffraction (and not by the paraxial equations of Chap. 2). The waist and focus are at different locations; in a weakly convergent beam, the separation may be large. The following equations allow calculation of the new waist size and location:

$$x' = \frac{-xf^2}{x^2 + \left(\dfrac{\pi w_1^{\,2}}{\lambda}\right)^2} \tag{6.32}$$

$$w_2^{\,2} = \frac{f^2 w_1^{\,2}}{x^2 + \left(\dfrac{\pi w_1^{\,2}}{\lambda}\right)^2} = w_1^{\,2}\left(\frac{x'}{-x}\right) \tag{6.33}$$

where w_1 = the radius (to the $1/e^2$ points) of the original waist
w_2 = the radius of the new waist formed by the optical system
f = the focal length of the lens
x = the distance from the first focal point of the lens to the plane of w_1
x' = the distance from the second focal point of the lens to the plane of w_2

Note that x and x' are usually negative and positive, respectively. Note also the similarity to the newtonian paraxial equation (Eq. 2.3).

Two points regarding the above are well worth emphasizing. First, laser researchers speak in terms of a "beam waist." Note that in the equations above and in common usage it is described as a radial dimension, not a diameter; the *diameter* of the waist is $2w$. Second, the waist and the focus are not the same thing, as a comparison of Eqs. 6.32 and 2.3 will indicate. In most circumstances the difference is trivial and gaussian beams may be handled by the usual paraxial equations. But when the beam convergence is small (i.e., with an *f*-number of a hundred or so), it is possible to distinguish both a focus and a separate beam waist. For example, if we project a 1-in laser beam (through a focusable beam expander) on a screen about 50 ft away, we can focus the beam to get the smallest possible spot on the screen. The focus is now at the screen. However, there is a location a few feet short of the screen at which a smaller beam diameter exists. This is the beam waist; it can be demonstrated by moving the screen (or a sheet of paper) toward the laser and observing the reduction of the spot size. Note that with the screen now at this beam waist position, the beam expander can be refocused to get a still smaller spot on the screen. Then there will be a new waist still closer to the laser, etc., etc., etc.

Note well that the *focus* is the smallest spot which can be produced on a surface at a given, fixed distance. The *waist* is the smallest diameter in the beam (see Gaskill, p. 435).

Note also that all the phenomena described in this section result from the gaussian distribution of beam intensity and *not* from the fact that the source may be a laser. The same effects could be produced by a radially graded filter placed over the aperture of the system. (The temporal and spatial coherence of a laser beam are, of course, what make it practical to demonstrate these effects.)

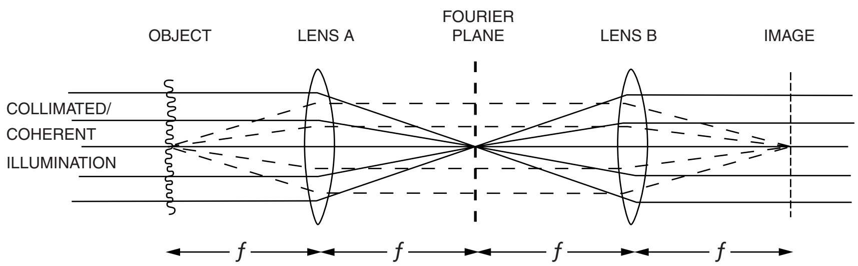

Figure 6.20

6.12 The Fourier Transform Lens and Spatial Filtering

In Fig. 6.20 we have a transparent object located at the first focal point of lens *A*. As indicated by the dashed rays in the figure, lens *A* images the object at infinity so that the rays originating at the axial point of the object are collimated. These rays are brought to a focus at the second focal plane of lens *B,* where the image of the object is located.

Now let us realize that Fourier theory allows us to consider the object as comprised of a collection of sinusoidal gratings of different frequencies, amplitudes, phases, and orientations. If our object is a simple linear grating with but a single spatial frequency, it will deviate the light through an angle α according to Eq. 6.24, except that a sinusoidal grating has but a single diffraction order, the first. Now, if the object is illuminated by collimated/coherent light, that diffracted light will be focused as two points in the second focal plane of lens *A* (which is indicated as the Fourier plane, midway between the lenses in Fig. 6.20). The points will be laterally displaced by $\delta = f \tan \alpha$ from the nominal focus. Thus, if an annular zone in the Fourier plane is obstructed, all the spatial information of the frequency corresponding to the radius of the obstruction will be removed (filtered) from the final image. Thus it can be seen that the Fourier plane constitutes a sort of map of the spatial frequency content of the object and that this content can be analyzed or modified in this plane.

Bibliography

Note: Titles preceded by an asterisk are out of print.

Campbell, J., and L. DeShazer, *J. Opt. Soc. Am.,* vol. 59, 1969, pp. 1427–1429.

Gaskill, J., *Linear Systems, Fourier Transforms, and Optics,* New York, Wiley, 1978.

Goodman, J., *Introduction to Fourier Optics,* New York, McGraw-Hill, 1968.

*Hardy, A., and F. Perrin, *The Principles of Optics,* New York, McGraw-Hill, 1932.

*Jacobs, D., *Fundamentals of Optical Engineering,* New York, McGraw-Hill, 1943.

Jenkins, F., and H. White, *Fundamentals of Optics,* New York, McGraw-Hill, 1976.

Kogelnick, H., in Shannon and Wyant (eds.), *Applied Optics and Optical Engineering,* vol. 7, New York, Academic, 1979.

Kogelnick, H., and T. Li, *Applied Optics,* 1966, pp. 1550–1567.

Pompea, S. M., and R. P. Breault, "Black Surfaces for Optical Systems," in *Handbook of Optics,* vol. 2, New York, McGraw-Hill, 1995, Chap. 37.

Silfvast, W. T., "Lasers," in *Handbook of Optics,* vol. 1, New York, McGraw-Hill, 1995, Chap. 11.

Smith, W., in W. Driscoll (ed.), *Handbook of Optics,* New York, McGraw-Hill, 1978.

Smith, W., in Wolfe and Zissis (eds.), *The Infrared Handbook,* Office of Naval Research, 1985.

Stoltzman, D., in Shannon and Wyant (eds.), *Applied Optics and Optical Design,* vol. 9, New York, Academic, 1983.

*Strong, J., *Concepts of Classical Optics,* New York, Freeman, 1958.

Walther, A., in Kingslake (ed.), *Applied Optics and Optical Engineering,* vol. 1, New York, Academic, 1965.

Exercises

1 Find the positions and diameters of the entrance and exit pupils of a 100-mm focal length lens with a diaphragm 20 mm to the right of the lens, if the lens diameter is 15 mm and the diaphragm diameter is 10 mm.

ANSWER: Entrance pupil is 25 mm to the right and 12.5 mm in diameter. Exit pupil is 20 mm to the right and 10 mm in diameter.

2 What is the relative aperture (*f*-number) of the lens of exercise #1 with light incident (a) from the left, and (b) from the right?

ANSWER: (a) $f/8$ (b) $f/10$

3 A telescope is composed of an objective lens, $f = 10$ in, diameter = 1 in and an eyelens, $f = 1$ in, dia. = $^1/_2$ in, which are 11 in apart. (a) Locate the entrance and exit pupils and find their diameters. (b) Determine the object and image fields of view in radians. Assume object and image to be at infinity.

ANSWER: (a) Entrance pupil is at the objective, diameter 1 in. Exit pupil is 1.1 in to the right of the eyelens and is 0.1 in diameter. (b) For zero vignetting, object field is ±0.01818 and image field is ±0.1818. For complete vignetting, object field is ±0.02727 and image field is ±0.2727.

4 A 4-in focal length *f*/4 lens is used to project an image at a magnification of four times ($m = -4$). What is the numerical aperture in object space and in image space?

ANSWER: NA = 0.1; NA = 0.025

5 An optical system composed of two thin elements forms an image of an object located at infinity. The front lens has a 16-in focal length, the rear lens an 8-in focal length, and the spacing between the two is 8 in. If the exit pupil is located at the rear lens and there is no vignetting, what is the illumination at an image point 3 in from the axis relative to the illumination on the axis?

ANSWER: 41 percent

6 A 6-in diameter *f*/5 paraboloid mirror is part of an infrared tracker which can tolerate a blur (due to defocusing) of 0.1 milliradians. (a) What tolerance must be maintained on the position of the reticle with respect to the focal point? (b) What is the tolerance if the system speed is *f*/2?

ANSWER: (a) ±0.015 in (b) ±0.0024 in

7 If the hyperfocal distance of a 10-in focal length, *f*/10 lens is 100 in, (a) what is the diameter of the acceptable blur spot, and (b) what is the closest distance at which an object is "acceptably" in focus? (c) Show that the answer to (b) is always one-half the hyperfocal distance.

ANSWER: (a) 0.111 in (b) 50 in

8 Compare the image illumination produced by an *f*/8 lens at a point 45° from the axis with that from an *f*/16 lens 30° off axis.

ANSWER: The *f*/16 is 56 percent of the *f*/8

9 Plot the illumination (in the manner of Fig. 6.16) in the diffraction pattern at the focus of a lens with a square aperture, (a) along a line through the axis at 90° to a side of the aperture, and (b) along a line at 45° (the diagonal) to the sides of the aperture.

10 An optical system is required to image a distant point source as a spot of 0.01 mm in diameter. Assuming that all the useful energy in the image spot will be within the first dark ring, what relative aperture (*f*-number) must the optical system have? Assume a wavelength of 0.00055 mm.

ANSWER: *f*/7.5

11 A pinhole camera has no lens but uses a very small hole some distance from the film to form its image. If we assume that light travels in straight lines, then the image of a distant point source will be a blur whose diameter is the same size as the pinhole. However, diffraction will spread the light into an Airy disk. Thus, the larger the hole, the larger the geometrical blur but the smaller the diffraction pattern. Assume that the sharpest picture will be produced when the geometrical blur is the same size as the central bright spot of the Airy disk. What size hole should be used when the film is 10 cm from the hole? (*Hint:* Equate the hole diameter to the diameter of the first dark ring of the Airy disk given by Eq. 6.20.)

ANSWER: 0.037 cm for $\lambda = 0.55\ \mu m$ (diameter = $\sqrt{2.44\lambda f}$)

12 What is the resolution limit (at the object) for a microscopic objective whose acceptance cone has a numerical aperture of (a) 0.25, (b) 0.8, (c) 1.2?

ANSWER: (a) 0.0013 mm, (b) 0.00042 nn, (c) 0.00028 mm

13 What diameter must a telescope objective have if the telescope is to resolve 11 seconds of arc? If the eye can resolve 1 minute of arc, what is the minimum power of the telescope?

ANSWER: 0.5 in; 5.5 ×

14 Compare the resolution of a prism and a grating. The prism has a 1-in base and its glass has a dispersion of 0.1 per micrometer. The grating is 1-in wide and is ruled with 15,000 lines per inch.

ANSWER: Prism resolution—2540; grating resolution—15,000 1st order, 30,000 2d order, etc.