Chapter

4

Prisms and Mirrors

4.1 Introduction

In most optical systems, prisms serve one of two major functions. In spectral instruments (spectroscopes, spectrographs, spectrophotometers, etc.) their function is to disperse the light or radiation; that is, to separate the different wavelengths. In other applications, prisms are used to displace, deviate, or reorient a beam of light or an image. In this type of use, the prism is carefully arranged so that it will *not* separate the different colors.

4.2 Dispersing Prisms

In a typical dispersing prism, as shown in Fig. 4.1, a light ray strikes the first surface at an angle of incidence I_1 and is refracted downward, making an angle of refraction I'_1 with the normal to the surface. The ray is thus deviated through an angle of $(I_1 - I'_1)$ at this surface. At the second surface the ray is deviated through an angle $(I'_2 - I_2)$, so the total deviation of the ray is given by

$$D = (I_1 - I'_1) + (I'_2 - I_2) \tag{4.1}$$

From the geometry of the figure it can be seen that angle I_2 is equal to $(A - I'_1)$, where A is the vertex angle of the prism; making this substitution in Eq. 4.1, we get

$$D = I_1 + I'_2 - A \tag{4.2}$$

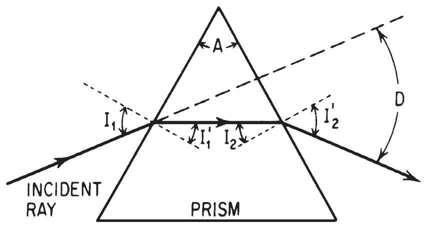

Figure 4.1 The deviation of a light ray by a refracting prism.

To compute the deviation produced by the prism we can readily determine the angles in Eq. 4.2 by Snell's law (Eq. 1.3) as follows (where n is the prism index):

$$\sin I'_1 = \frac{1}{n} \sin I_1 \tag{4.3}$$

$$I_2 = A - I'_1 \tag{4.4}$$

$$\sin I'_2 = n \sin I_2 \tag{4.5}$$

While it is ordinarily much more convenient to calculate the deviation step by step, using the equations above, it is possible to combine them into a single expression for D, in terms of I_1, A, and n as follows:

$$D = I_1 - A + \arcsin\left[(n^2 - \sin^2 I_1)^{1/2} \sin A - \cos A \sin I_1 \right] \tag{4.6}$$

It is apparent that the deviation is a function of the prism index and that the deviation will be increased as the index is raised. For optical materials, the index of refraction is higher for short wavelengths (blue light) than for long wavelengths (red light). Therefore, the deviation angle will be greater for blue light than red, as indicated in Fig. 4.2. This variation of the deviation angle with wavelength is called the dispersion of the prism. An expression for the dispersion can be found by differentiating the preceding equations with respect to the index n, assuming that I_1 is constant, yielding,

$$dD = \frac{\cos I_2 \tan I'_1 + \sin I_2}{\cos I'_2}\, dn \tag{4.7}$$

The angular dispersion with respect to wavelength is simply $dD/d\lambda$ and is obtained by dividing both sides of Eq. 4.7 by $d\lambda$. The resulting $dn/d\lambda$ term on the right is the index dispersion of the prism material.

4.3 The "Thin" Prism

If all the angles involved in the prism are very small, we can, as in the paraxial case for lenses, substitute the angle itself for its sine. This case occurs when the prism angle A is small and when the ray is

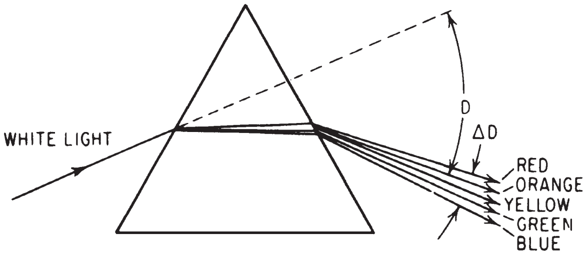

Figure 4.2 The dispersion of white light into its component wavelengths by a refracting prism (highly exaggerated).

almost at normal incidence to the prism faces. Under these conditions, we can write

$$i'_1 = \frac{i_1}{n}$$

$$i_2 = A - i'_1 = A - \frac{i_1}{n}$$

$$i'_2 = ni_2 = nA - i_1$$

$$D = i_1 + i'_2 - A = i_1 + nA - i_1 - A$$

and finally

$$D = A\,(n - 1) \tag{4.8a}$$

If the prism angle A is small but the angle of incidence I is *not* small, we get the following approximate expression for D (which neglects powers of I larger than 3).

$$D = A\,(n - 1)\left[1 + \frac{I^2\,(n + 1)}{2n} + \cdots\right] \tag{4.8b}$$

These expressions are of great utility in evaluating the effects of a small prismatic error in the construction of an optical system since it allows the resultant deviation of the light beam to be determined quite readily.

The dispersion of a "thin" prism is obtained by differentiating Eq. 4.8a with respect to n, which gives $dD = Adn$. If we substitute A from Eq. 4.8a, we get

$$dD = D\,\frac{dn}{(n - 1)} \tag{4.9}$$

Now the fraction $(n - 1)/\Delta n$ is one of the basic numbers used to characterize optical materials. It is called the reciprocal relative dispersion, Abbe V number, or V-value. Ordinarily n is taken as the index for the helium d line (0.5876 μm) and Δn is the index difference between the hydrogen F(0.4861 μm) and C(0.6563 μm) lines, and the V-value is given by

$$V = \frac{n_d - 1}{n_F - n_C} \tag{4.10}$$

Making the substitution of $1/V$ for $dn/(n - 1)$ in Eq. 4.9, we get

$$dD = \frac{D}{V} \tag{4.11}$$

which allows us to immediately evaluate the chromatic dispersion produced by a thin prism.

4.4 Minimum Deviation

The deviation of a prism is a function of the initial angle of incidence I_1. It can be shown that the deviation is at a minimum when the ray passes symmetrically through the prism. In this case $I_1 = I'_2 = ½(A + D)$ and $I'_1 = I_2 = A/2$, so that if we know the prism angle A and the minimum deviation angle D_0 it is a simple matter to compute the index of the prism from

$$n = \frac{\sin I_1}{\sin I'_1} = \frac{\sin \frac{1}{2}(A + D_0)}{\sin \frac{1}{2}A} \tag{4.12}$$

This is a widely used method for the precise measurement of index, since the minimum deviation position is readily determined on a spectrometer. This position for the prism is also approximated in most spectral instruments because it allows the largest diameter beam to pass through a given prism and also produces the smallest amount of loss due to surface reflections.

4.5 The Achromatic Prism and the Direct Vision Prism

It is occasionally useful to produce an angular deviation of a light beam without introducing any chromatic dispersion. This can be done by combining two prisms, one of high-dispersion glass and the other of low-dispersion glass. We desire the sum of their deviations to equal $D_{1,2}$ and the sum of their dispersions to equal zero. Using the equations for "thin" prisms (Eqs. 4.8 and 4.11), we can express these requirements as follows:

$$\text{Deviation } D_{1,2} = D_1 + D_2 = A_1(n_1 - 1) + A_2(n_2 - 1)$$

$$\text{Dispersion } dD_{1,2} = dD_1 + dD_2 = 0 = \frac{D_1}{V_1} + \frac{D_2}{V_2}$$
$$= \frac{A_1(n_1 - 1)}{V_1} + \frac{A_2(n_2 - 1)}{V_2}$$

A simultaneous solution for the angles of the two prisms gives

$$A_1 = \frac{D_{1,2}V_1}{(n_1 - 1)(V_1 - V_2)}$$
$$A_2 = \frac{D_{1,2}V_2}{(n_2 - 1)(V_2 - V_1)} \tag{4.13}$$

It is apparent that the prism angles will have opposite signs and that the prism with the larger V-value (smaller relative dispersion) will have the larger angle. A sketch of an achromatic prism is shown in Fig. 4.3. Note that the emerging rays are not coincident but are parallel, indicating the same angular deviation.

In the *direct vision prism* it is desired to produce a dispersion without deviating the ray. By setting the deviation $D_{1,2}$ equal to zero and preserving the dispersion term $dD_{1,2}$ in the preceding equations we can solve for the angles of two prisms which will produce the desired result. The solution is

$$A_1 = \frac{dD_{1,2}V_1V_2}{(n_1 - 1)(V_2 - V_1)}$$
$$A_2 = \frac{dD_{1,2}V_1V_2}{(n_2 - 1)(V_1 - V_2)} \tag{4.14}$$

A two-element direct vision prism is shown in Fig. 4.4a. In order to obtain a large enough dispersion for practical purposes it is often necessary to use more than two prisms. Figure 4.4b shows the application of such a prism to a hand spectroscope.

Since Eqs. 4.13 and 4.14 were derived using the equations for thin prisms, it is obvious that the values of the component prism angles

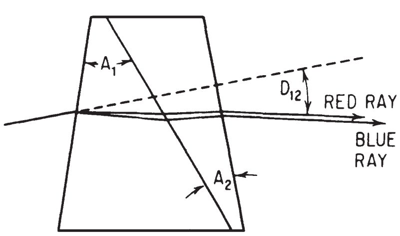

Figure 4.3 An achromatic prism. The red and blue rays emerge parallel to each other; no chromatic dispersion is introduced by the deviation.

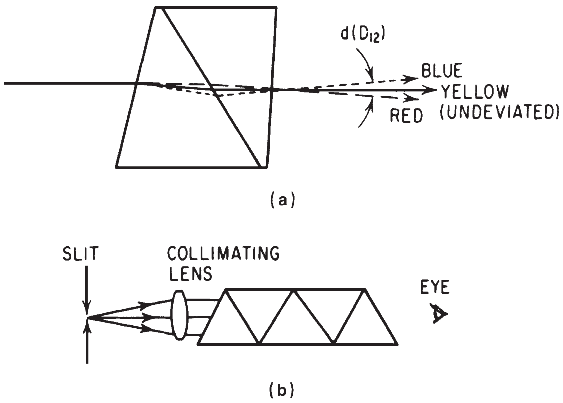

Figure 4.4 (a) A direct vision prism disperses the light into its spectral components without deviation of the beam. (b) Hand spectroscope. The collimating lens produces a magnified image of the slit at infinity for easy viewing. The prism then disperses the light into a spectrum without deviation of the yellow ray.

which they give will be approximations to the exact values when the prisms are other than thin. For exact work, these approximate values must be adjusted by exact ray tracing based on Snell's law.

4.6 Total Internal Reflection

When a light ray passes from a higher index medium to one with a lower index, the ray is refracted away from the normal to the surface as shown in Fig. 4.5a. As the angle of incidence is increased, the angle of refraction increases at a greater rate, in accordance with Snell's law ($n > n'$):

$$\sin I' = \frac{n}{n'} \sin I$$

When the angle of incidence reaches a value such that $\sin I = n'/n$, then $\sin I' = 1.0$ and $I' = 90°$. At this point none of the light is transmitted through the surface; the ray is totally reflected back into the denser medium, as is any ray which makes a greater angle to the normal. The angle

$$I_c = \arcsin \frac{n'}{n} \tag{4.15}$$

is called the *critical angle* and for an ordinary air-glass surface has a value of about 42° if the index of the glass is 1.5; for an index of 1.7, the critical angle is near 36°; for an index of 2.0, 30°; for an index of 4.0, 14.5°.

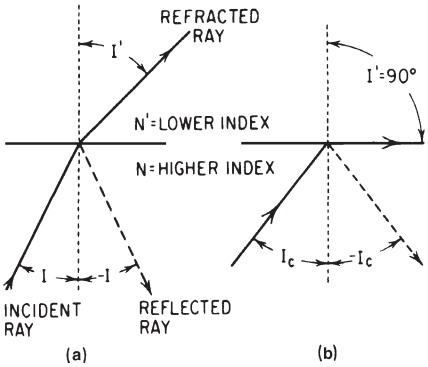

Figure 4.5 Total internal reflection occurs when a ray, passing from a higher to a lower index of refraction, has an angle of incidence whose sine equals or exceeds n'/n.

For practical purposes, if the boundary surface is smooth and clean, 100 percent of the energy is redirected along the totally reflected ray. However, it should be noted that the electromagnetic field associated with the light actually does penetrate the surface for a relatively short (to the order of a wavelength) distance. If there is anything near the other side of the boundary surface, the total internal reflection can be "frustrated" to some extent and a portion of the energy will be transmitted. Since the distance of effective penetration is only to the order of the wavelength of the light involved, this phenomenon has been used as the basis of a light valve, or modulator. In the German "Licht-Sprecher," an external piece of glass was placed in contact with the reflecting face of a prism to frustrate the reflection, and then moved an extremely short distance away (e.g., a few micrometers) to reinstate the reflection.

It should also be noted that the reflection of a totally reflecting surface is *decreased* by aluminizing or silvering the surface. When this is done, the reflectance drops from 100 percent to the reflectance of the coating applied to the surface.

4.7 Reflection from a Plane Surface

Since the prism systems which are discussed in the balance of this chapter are primarily reflecting prisms (the majority of which can be replaced by a system of plane mirrors), we shall first discuss the imaging properties of a plane reflecting surface. Rays originating at an object are reflected according to the law of reflection, which states that both the incident and reflected rays lie in the plane of incidence and that both rays make equal angles with the normal to the surface. The normal to the surface is the perpendicular at the point where the ray strikes the surface, and the plane of incidence is that plane containing the incident ray and the normal.

In Fig. 4.6, the plane of the page is the plane of incidence. Two rays from point *P* are shown reflected from the surface *MM′*. By extending the rays backward, it can be seen that after reflection they appear to be coming from point *P′*, which is a virtual image of point *P*. Both *P* and *P′* lie on the same normal to the surface (*POP′*), and the distance *OP* is exactly the same as the distance *OP′*.

If we now consider an extended object such as the arrow *AB* in Fig. 4.7, we can readily locate the position of its image by using the principles of the preceding paragraph to locate the images of points *A* and *B*. An observer at *E* looking *directly* at the arrow would see the arrowhead *A* at the top of the arrow. However, in the reflected image, the arrowhead (*A′*) is at the bottom of the arrow. The image of the arrow has been reoriented (or inverted) by the reflection.

If we add a crosspiece *CD* to the arrow, the image is formed as shown in Fig. 4.8, and although the image of the arrow has been inverted, the image of the crosspiece has the same left-to-right orientation as the object.

The preceding discussion has treated reflection from the standpoint of an observer viewing a reflected image. Since the path of light rays is completely reversible, we can equally well consider point *P′* in Fig. 4.6 to be an image formed by a lens at the right. Then *P* would be the reflected image of *P′*. Similarly in Figs. 4.7 and 4.8, we may replace the eye with a lens whose image is the primed figure (*A′B′* or *A′B′C′D′*) and view the unprimed figures as their reflected images.

A point worth noting is that reflection constitutes a sort of "folding" of the ray paths. In Fig. 4.9, the lens images the arrow at *AB*. If we now insert reflecting surface *MM′*, the reflected image is at *A′B′*. Notice that if the page were folded along *MM′*, the arrow *AB* and the solid line rays would exactly coincide with the arrow *A′B′* and the reflected (dashed) rays. It is frequently convenient to "unfold" a complex reflecting system; one advantage of this device is that an accurate drawing of the ray paths becomes a simple matter of straight lines.

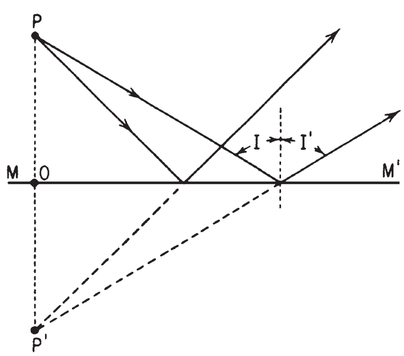

Figure 4.6 A plane reflecting surface forms a virtual image of an object point. Object and image are equidistant from the reflecting surface, and both lie on the same normal to the surface.

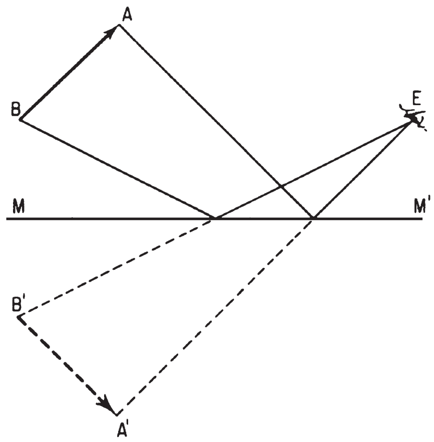

Figure 4.7 The reflected image *A′B′* of the arrow *AB* appears inverted to an observer at *E*.

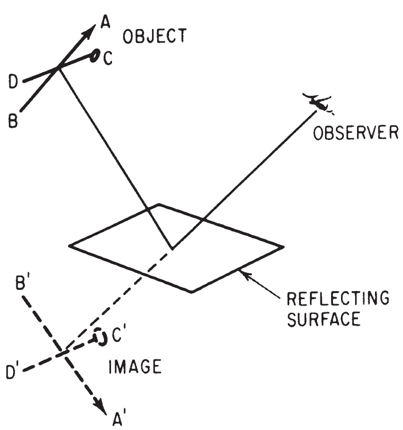

Figure 4.8 The reflected image is inverted top to bottom, but not left to right.

A useful technique to determine the image orientation after passage through a system of reflectors is to imagine that the image is a transverse arrow, or pencil, which is bounced off the reflecting surface, much as a thrown stick would be bounced off a wall. Figure 4.10 illustrates the technique. The first illustration shows the pencil approaching and striking the reflecting surface, the second shows the point bouncing off the reflector and the blunt end continuing in the original direction, and the third shows the pencil in the new orientation after the reflection. If the process is repeated with the pencil perpendicular to the plane of the paper, the orientation of the other meridian of the image can be determined. The procedure can then be repeated through each reflection in the system.

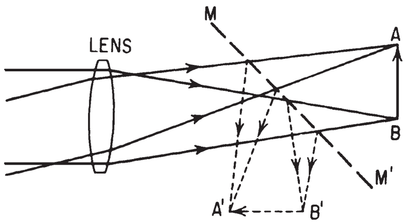

Figure 4.9 The reflecting surface *MM′* folds the optical system. Note that if the page is folded along *MM′*, the rays and images coincide.

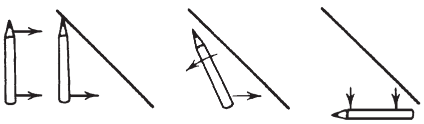

Figure 4.10 A useful technique in determining the orientation of a reflected image is to visualize the image as a pencil "bouncing" off a solid wall as it moves along the system axis.

A card marked with the arrow and crossbar of Fig. 4.11 is also useful for this purpose. The reader's attention is directed to the fact that the initial orientation of the pencil, or pattern, is chosen so that one meridian of the pattern coincides with the plane of incidence. In the majority of reflecting systems, one or the other of the meridians will be in the plane of incidence throughout the system, and the application of this technique is straightforward. Where this is not the case, the card can be marked with a second set of meridians so that the second set is aligned with the plane of incidence. This second set can then be carried through the reflection as before; the orientation of the final image is of course given by the original set of markings. Figure 4.20b exemplifies this method.

4.8 Plane Parallel Plates

As will become apparent, most prism systems are the equivalent of a thick block of glass. Thus we continue with a discussion of the effects produced by a plane-parallel plate of glass. Figure 4.12 shows a lens which, in air, would form an image at *P*. The insertion of the plane parallel plate between the lens and *P* displaces the image to *P′*. If we trace the path of the light rays through the plate, we first notice that the ray emerging from the plate has exactly the same slope angle that it had before passing through the plate, since by Snell's law, $\sin I'_1 = (1/n) \sin I_1$, and $I_2 = I'_1$ (since the surfaces are parallel). Thus, $\sin I_2 = \sin I'_1 = (1/n) \sin I_1 = (1/n) \sin I'_2$, and $I_1 = I'_2$. Therefore, the effective focal length of the lens system, and the size of the image, are unchanged by the insertion of the plate.

The amount of longitudinal displacement of the image is readily determined by application of the paraxial raytracing formulas of Chap. 2, and is equal to $(n - 1)t/n$. The effective thickness of the plate compared to air (the equivalent air thickness) is less than the actual thickness t

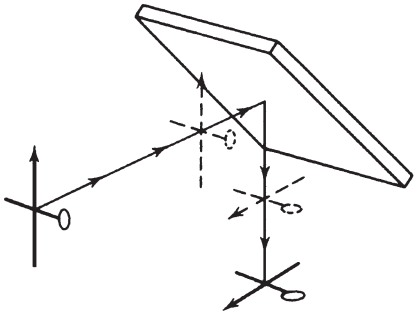

Figure 4.11 Image orientation after reflection.

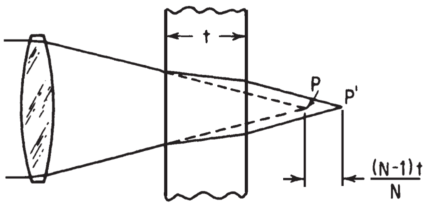

Figure 4.12 The longitudinal displacement of an image by a plane parallel glass plate.

by the amount of this shift. The *equivalent air thickness* is thus found by subtracting the displacement from the thickness and is equal to t/n. The concept of equivalent thickness is useful when one wishes to determine whether a certain size prism can be fitted into the available air space of an optical system, and also in prism system design.

If the plate is rotated through an angle I as shown in Fig. 4.13, it can be seen that the "axis ray" is laterally displaced by an amount D, which is given by

$$D = t \cos I (\tan I - \tan I') = t \frac{\sin (I - I')}{\cos I'}$$

or

$$D = t \sin I \left(1 - \frac{\cos I}{n \cos I'}\right)$$

or

$$D = t \sin I \left[1 - \sqrt{\frac{1 - \sin^2 I}{n^2 - \sin^2 I}}\right]$$

A power series expansion yields the following expression:

$$D = \frac{tI (n - 1)}{n} \left[1 + \frac{I^2 (-n^2 + 3n + 3)}{6n^2} + \frac{I^4 (n^4 - 15n^3 - 15n^2 + 45n + 45)}{120n^4} + \cdots \right]$$

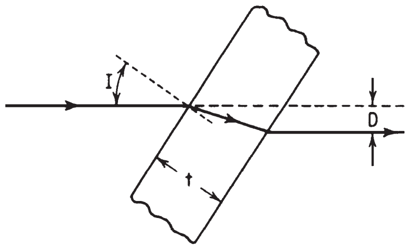

Figure 4.13 The lateral displacement of a ray by a tilted plane parallel plate.

For small angles, we can make the usual substitution of the angle for its sine or tangent, or simply use the first term of the expansion to get

$$d = \frac{ti\,(n - 1)}{n}$$

This lateral displacement by a tilted plate is used in high-speed cameras (where the rotating plate displaces the image an amount approximately equal to the travel of the continuously moving film) and in optical micrometers. The optical micrometer is usually placed in front of a telescope and used to displace the line of sight. The amount of displacement is read off a calibrated drum connected to the mechanism which tilts the plate.

When used in parallel light, a plane parallel plate is free of aberrations (since the rays enter and leave at the same angles). However, if the plate is inserted in a convergent or divergent beam, it does introduce aberrations. The longitudinal image displacement $(n - 1)t/n$ is greater for short wavelength light (higher index) than for long, so that overcorrected chromatic aberration is introduced. The amount of displacement is also greater for rays making large angles with the axis; this is, of course, overcorrected spherical aberration. When the plate is tilted, the image formed by the meridional rays is shifted backward while the image formed by the sagittal rays (in a plane perpendicular to the page in the figures) is shifted by a lesser amount, so that astigmatism is introduced.

The amount of aberration introduced by a plane parallel plate can be computed by the formulas below. Reference to Fig. 4.14 will indicate the meanings of the symbols

U and u—slope angle of the ray to the axis

U_p and u_p—the tilt of the plate

t—thickness of the plate

n—index of the plate

V—Abbe V number $(n_d - 1)/(n_F - n_C)$

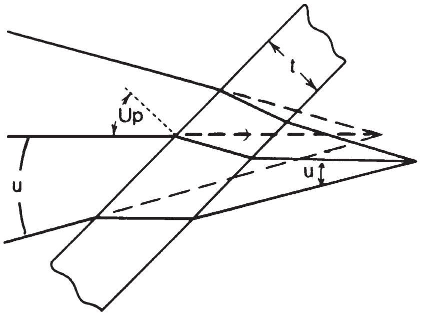

Figure 4.14

$$\text{Chromatic aberration} = l'_F - l'_C = \frac{t(n-1)}{n^2 V}$$

$$\text{Spherical aberration} = L' - l' = \frac{t}{n}\left[1 - \frac{n \cos U}{\sqrt{n^2 - \sin^2 U}}\right] \qquad \text{(exact)}$$

$$= \frac{tu^2 (n^2 - 1)}{2n^3} \qquad \text{(third order)}$$

$$\text{Astigmatism} = (l'_s - l'_t) = \frac{t}{\sqrt{n^2 - \sin^2 U_p}} \times \left[\frac{n^2 \cos^2 U_p}{(n^2 - \sin^2 U_p)} - 1\right] \qquad \text{(exact)}$$

$$= \frac{-tu_p^{\,2} (n^2 - 1)}{n^3} \qquad \text{(third order)}$$

$$\text{Sagittal coma} = \frac{tu^2 u_p (n^2 - 1)}{2n^3} \qquad \text{(third order)}$$

$$\text{Lateral chromatic} = \frac{tu_p (n-1)}{n^2 V} \qquad \text{(third order)}$$

These expressions are extremely useful in estimating the effect that the introduction (or removal) of a plate or a prism system will have on the state of correction of an optical system.

A common use for a glass plate is as a beam splitter, tilted at an angle of 45°. In this orientation the astigmatism is approximately a quarter of the thickness of the plate. Since this can severely degrade the image, such plate beam splitters are not recommended in convergent or divergent beams (i.e., where u in Fig. 4.14 is nonzero). Note that the astigmatism can be nullified by inserting another identical plate which is tilted in a meridian 90° to the original plate, by introducing either a weak cylinder or a tilted spherical surface, or by wedging the plate.

4.9 The Right-Angle Prism

The right-angle prism, with angles of 45°–90°–45°, is the building block of most nondispersing prism systems. Figure 4.15 shows a parallel bundle of rays passing through such a prism, entering through one face, reflecting from the hypotenuse face, and leaving through the second face. If the rays are normally incident on the face of the prism, they are deviated through an angle of 90°. At the hypotenuse face, the rays have an angle of incidence of 45° so that they are subject to total internal reflection. If the entrance and exit faces are low-reflection-coated, this makes the prism a highly efficient reflector for visual usage since the only losses are the absorption of the material and the reflection losses at the faces which total a few percent or less. (In the ultraviolet and infrared portions of the spectrum, the absorption of a prism may be quite objectionable.) It can be seen that the total internal reflection is limited to rays which have angles of incidence greater than the critical angle, and many prism systems are made of high-index glass to permit total reflection over larger angles.

By *unfolding* the prism, as indicated by the dashed lines in Fig. 4.16, it is apparent that the prism is the equivalent of a glass block with parallel faces, with a thickness equal to the length of the entrance or exit faces. The equivalent air thickness of the block is, of course, this thickness divided by the index of the prism.

If the 45°–90°–45° prism is used with the light beam incident on the hypotenuse face as shown in Fig. 4.17, the light is totally reflected twice and the rays emerge in the opposite direction, having been deviated through 180°. Figure 4.17 also indicates the unfolded prism path and the image orientation of this prism. Notice that the image has

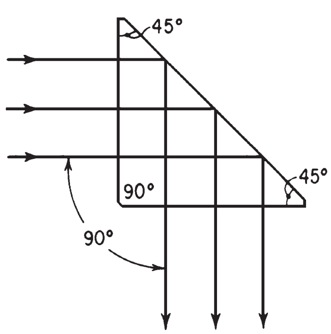

Figure 4.15 Right-angle prism.

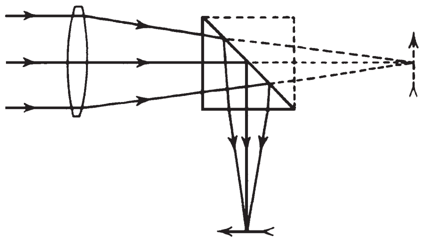

Figure 4.16

been inverted, top to bottom, but not left to right. The unfolded prism path is called a *tunnel diagram.* Such a diagram can be used to determine the angular field of the prism as well as the size of the beam which will pass through the prism.

Used in this way, this prism is a *constant-deviation prism.* Regardless of the angle at which a ray enters the prism, the emergent ray will be parallel, as shown in Fig. 4.18a. This characteristic is a property of the two reflecting surfaces of the prism. A system which directs the light ray back on itself is called a retrodirector; this prism is a retrodirector in one meridian only. (Another of the many constant-deviation systems possible with two reflectors is the 90° deviation arrangement shown in Fig. 4.18b, where the reflecting surfaces are at 45° to each other.) The constant-deviation angle is just twice the angle between the two mirrors.

A prism made by cutting off one corner of a cube, so that there are three mutually perpendicular reflecting surfaces, is retrodirective in both meridians. The corner cube (or cube corner) reflector will return all the light rays striking it back toward their source, although the rays will be displaced laterally.

A third orientation of the 45°–90°–45° prism is shown in Fig. 4.19, in which the bundle of rays arrives parallel to the hypotenuse face of the prism. After being refracted downward at the entrance face, the rays are reflected upward from the hypotenuse and emerge after a second refraction at the exit face. The unfolded path of the rays (shown in dashed lines) indicates that this prism is the equivalent of a plane parallel plate which is tilted with respect to the axis of the bundle, whereas in the preceding examples the prism faces have been normal to the axis. If this prism is used in a convergent light beam, it will introduce a substantial amount of astigmatism (roughly equal to one-quarter of its thickness). For this reason, this prism, which is known as a *Dove prism,* is used almost exclusively in parallel light. Since the apex of the prism is not used by the light beam, the prism is usually truncated at AA'.

The Dove prism has a very interesting effect on the orientation of the image. In Fig. 4.20a, the arrow and crossbar pattern is shown to be inverted from top to bottom but not left to right. If the prism is

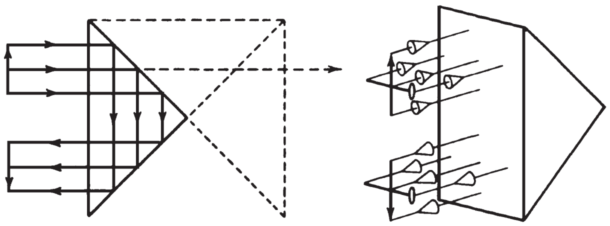

Figure 4.17 Right-angle prism used with hypotenuse as entrance and exit face.

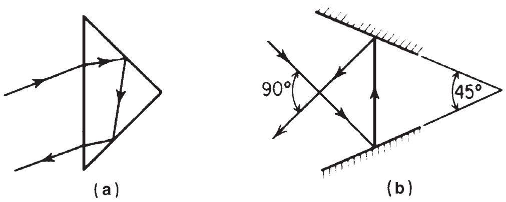

Figure 4.18 (a) The right-angle prism used in the manner shown is a constant-deviation prism, in that each ray is reflected through exactly 180°. The entering and emergent paths are parallel, regardless of the initial angle the ray makes with the prism. (b) A pair of constant-deviation mirrors. In this case, the deviation produced by the two reflections is always exactly 90°.

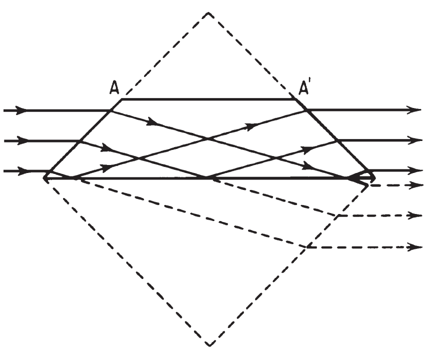

Figure 4.19 The Dove prism. The dashed lines show that the Dove prism is equivalent to a tilted plate and will introduce astigmatism when used in convergent or divergent beams.

rotated 45°, as in Fig. 4.20b, the image is rotated through 90°; if the prism is rotated 90° as in Fig. 4.20c, the pattern is rotated 180°. Thus, the image is rotated twice as fast as the prism. (The analysis of the image orientation in Fig. 4.20b is an example of the use of an auxiliary pattern as described in Sec. 4.7. The auxiliary pattern is shown in dotted lines in Fig. 4.20b.)

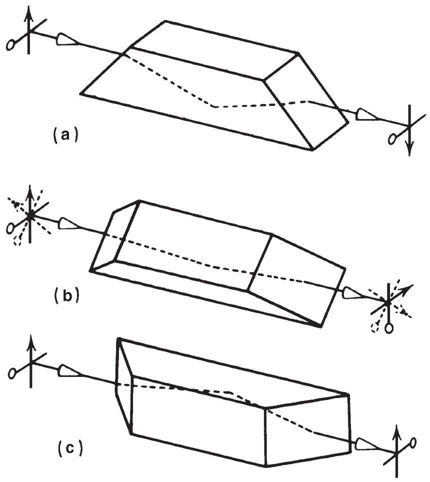

Figure 4.20 The orientation of an image by a Dove prism. (a) Original position. (b) Prism rotated 45°; image is rotated 90° (c) Prism rotated 90°; image is rotated 180°. Note that the dotted arrow and crossbar in (b) is oriented so that the dotted arrow is in the plane of incidence to simplify the analysis of the image orientation.

The length of the Dove prism is four to five times the diameter of the bundle of rays which it will transmit. If two Dove prisms are cemented hypotenuse to hypotenuse (after silvering or aluminizing these faces), the aperture is thereby doubled with no increase in length. The double Dove prism is used in parallel light as is the Dove. It must be precisely fabricated to avoid producing two slightly separated images. When the double Dove is rotated, or tipped, about its center, it can be used as a scanner to change the direction of sight of a telescope or periscope.

4.10 The Roof Prism

If the hypotenuse face of a right-angle prism is replaced by a "roof," i.e., two surfaces at 90° whose intersection lies in the hypotenuse, the prism is called a *roof,* or *Amici, prism.* Face and side views of a roof prism are shown in Fig. 4.21. The addition of the roof to the prism serves to introduce an extra inversion to the image, as can be seen by comparing the final orientation of the cross bar in Fig. 4.11 with that in Fig. 4.22a. This can be understood by tracing the path of the dashed ray in Fig. 4.22a which connects the circles in the arrow and crossbar figures before and after passing through the prism.

The angle of incidence (at the roof surface) of the ray shown in Fig. 4.22a is about 60° instead of the 45° it would be for the same ray in the right-angle prism. Even a ray perpendicular to the roof edge has an angle of incidence of 45°. The result is that a roof surface allows total internal reflection for beam angles which would be transmitted through the hypotenuse face of a right-angle prism.

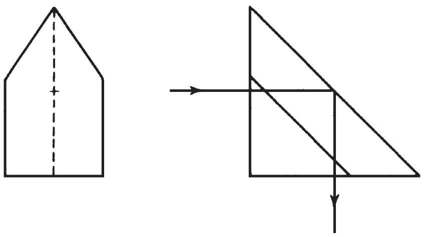

Figure 4.21 Roof, or Amici, prism.

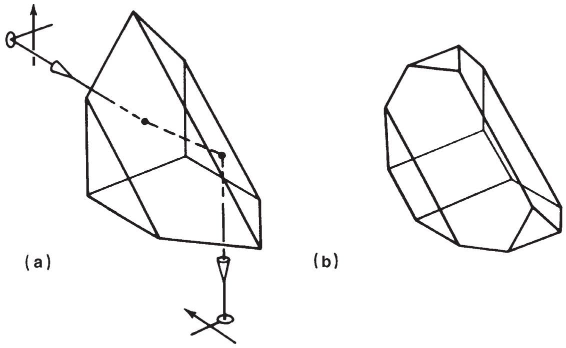

Figure 4.22 Amici prism (a) showing a single ray path through the prism and indicating the image orientation, (b) with truncated corners to reduce weight without sacrifice of useful aperture.

In practice, the Amici prism is usually fabricated with the corners cut off, as shown in Fig. 4.22b, in order to reduce the size and weight of the prism. The 90° roof angle must be made to a high order of accuracy. If there is an error in the roof angle, the beam is split into two beams which diverge at an angle which is six times the error. Thus, to avoid any apparent doubling of the image, the roof angle is usually made accurate to one or two seconds of arc.

The introduction of a roof degrades the diffraction-limited resolution by a factor approaching 2 in the direction perpendicular to the roof edge (due to a polarization/phase shift on reflection) no matter how perfectly the prism is made. Multilayer coatings have been developed which will reduce this effect.

4.11 Erecting Prism Systems

In an ordinary telescope, the objective lens forms an inverted image of the object, which is then viewed through the eyepiece. The image seen by the eye is upside down and reversed from left to right, as indicated in Fig. 4.23. To eliminate the inconvenience of viewing an inverted image, an erecting system is often provided to re-invert the image to its proper orientation. This may be a lens system or a prism system.

Porro prism of the first type

The most commonly used prism-erecting system is the Porro prism of the first type, illustrated in Fig. 4.24. The Porro system consists of two right-angle prisms oriented at 90° to each other. The first prism inverts the image from top to bottom and the second prism reverses it from left to right. The optical axis is displaced laterally, but is not deviated. One can see that if this system is inserted into the telescope of Fig. 4.23, the final image will have the same orientation as the object. Although the prism system is ordinarily inserted between the objective and eyepiece (to minimize its size), it will erect the image regardless of where it is placed in the system.

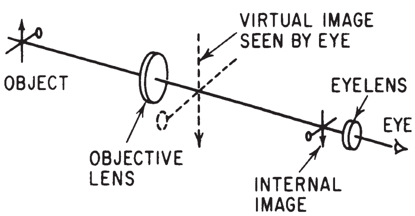

Figure 4.23 In a simple telescope, the objective lens forms a real, inverted internal image of the object, which is reimaged by the eyelens. The image seen by the eye is a virtual inverted image of the object.

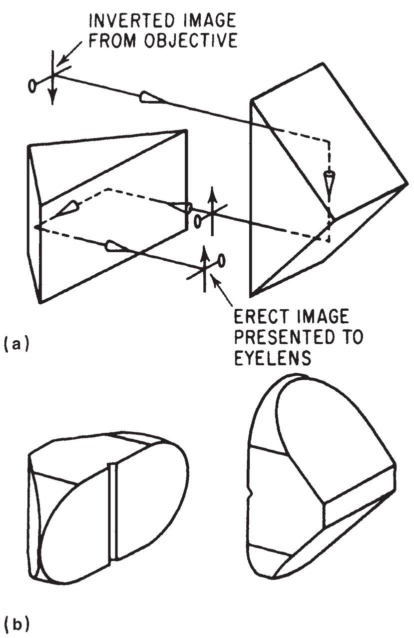

Figure 4.24 Porro prism system (first type) (a) indicating the way the Porro system erects an inverted image. (b) Porro prisms are usually fabricated with rounded ends to save space and weight. Note that the spacing between the prisms has been shown increased for clarity.

The Porro prism (first type) owes its popularity to the fact that the 45°–90°–45° prisms are relatively easy and inexpensive to manufacture, with no critical tolerances. However, if the prisms are not mounted so that their roof edges are exactly at 90° to each other, the final image will be rotated through twice the angular mounting error. This is of special importance in binocular systems where the image presented to one eye must be identical to that presented to the other.

A shallow ground slot is often cut across the center of the hypotenuse face of each prism to prevent unwanted grazing angle reflections from this face which originate from outside the field of view. See also Fig. 4.39.

Porro prism of the second type

The Porro prism of the second type is shown in Fig. 4.25, and serves the same purpose as the Porro #1 system. Both Porro systems function by total internal reflection so that no silvering is required. It is common to round off the ends of the prisms to conserve space and weight.

The second Porro is somewhat more difficult to fabricate than the first type, but in some applications its compactness, and the fact that the prisms can be readily cemented together, offer compensating advantages. The Porro #2 may also be made in three pieces, by cementing two small right-angle prisms on the hypotenuse of a large right-angle prism as indicated in Fig. 4.25b. The lateral displacement of the axis is less than that for the Porro #1 system.

Abbe prism

The Abbe (or Koenig, or Brashear-Hastings) prism (Fig. 4.26) is an erecting prism which can be used when it is desired to erect the image

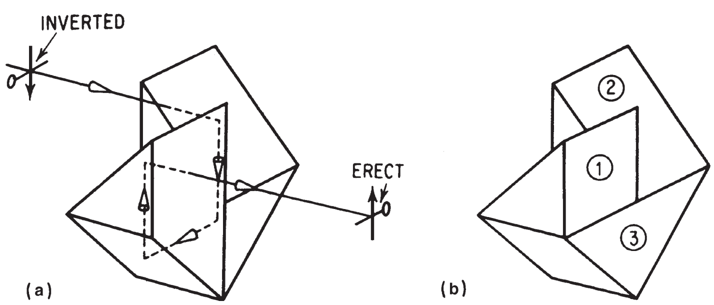

Figure 4.25 Porro prism system (second type) (a) indicating the erection of an inverted image. This system is shown made from two prisms in (a) and from three prisms in (b).

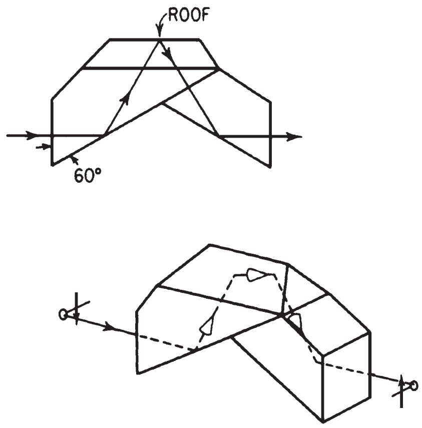

Figure 4.26 Abbe prism. Used as an in-line erecting system, it does not displace the axis as the Porro systems do, nor does it materially displace the image longitudinally.

without displacing the axis as the Porro prisms do. The roof is necessary to provide the left-to-right reversal of the image; the roof angle must be made accurately to avoid image doubling.

If this prism is made without the roof, it will invert the image in one meridian only, just as the Dove prism. However, since its entrance and exit faces are normal to the system axis, it may be used in a converging beam without introducing astigmatism.

Other erecting prisms

Among the many prisms designed to erect an image are those sketched in Fig. 4.27. The fact that the image is inverted and reversed left to right after passing through these prisms may be verified by the methods outlined in Sec. 4.7. Notice that each prism (except Fig. 4.27f) has been arranged so that the axial ray enters and leaves the prism normal to the prism faces and that all reflections are total internal reflections. In the Leman and Goerz prisms, the axis is displaced but not deviated. In the Schmidt and modified Amici prisms, the axis is deviated through a definite angle, which can be selected by the designer (within the limits allowed by total internal reflection). Note also that the roof surface is used at the location where the angle of incidence is small and where there would be light leakage through an ordinary surface.

4.12 Inversion Prisms

The Dove prism (Figs. 4.19 and 4.20) and the roofless Abbe prism mentioned in Sec. 4.11 are examples of prisms which invert the image in one meridian but not the other. The plane mirror and the right-angle prism (Figs. 4.11 and 4.16) are also simple inversion systems. Figure

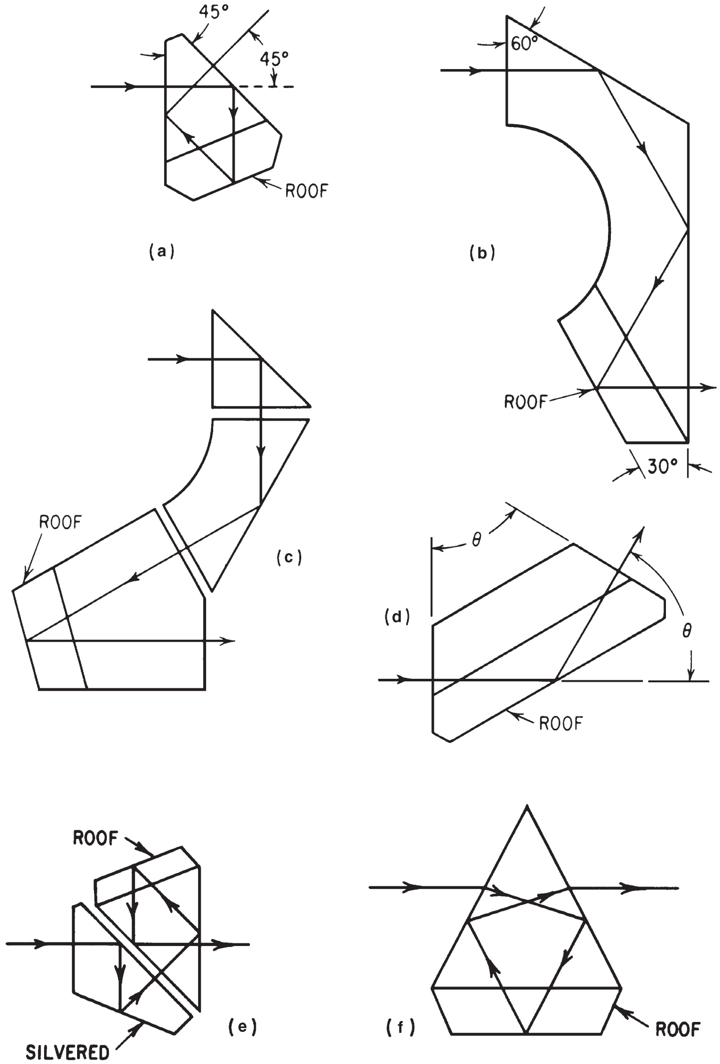

Figure 4.27 Erecting prisms: (a) Schmidt prism; (b) Leman (or Sprenger) prism; (c) Goerz prism; (d) modified Amici prism; (e) roofed Pechan prism; (f) roofed delta prism.

4.28 shows the above prisms plus the Pechan prism, which is a relatively compact prism for this purpose. Notice that the addition of a "roof" to any of these prism swill convert it to an erecting system.

An inversion prism is also known as a *derotation prism,* since all inversion prisms rotate the image in the same manner as the Dove prism, as shown in Fig. 4.20.

The mirror version of Fig. 4.28b is called a *k-mirror* and is useful in infrared and ultraviolet applications where material for a solid prism system is impractical.

4.13 The Penta Prism

The Penta prism (Fig. 4.29a) will neither invert nor reverse the image. Its function is to deviate the line of sight by 90°. It has the valuable property of being a constant-deviation prism, in that it deviates the line of sight through the same angle regardless of its orientation to the line of sight.

Most of the prism systems described in this chapter could be replaced by a series of plane mirrors, and this is sometimes done for reasons of weight and/or economy. However, a prism, as a monolithic glass block, is a very stable system and is not as subject to environmental variation of angles as is an assemblage of mirrors on a metal support block.

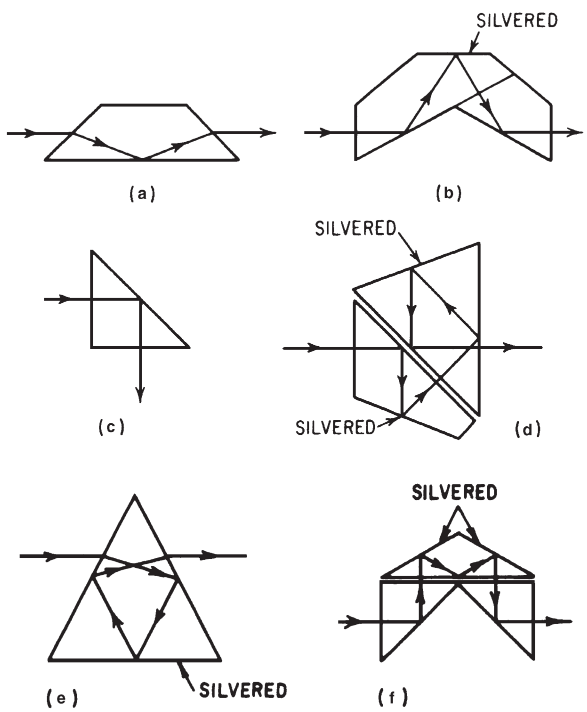

Figure 4.28 Inversion (or derotation) prisms: (a) Dove prism; (b) reversion prism; (c) right-angle prism; (d) Pechan prism; (e) delta, or Taylor, prism; (f) compact prism.

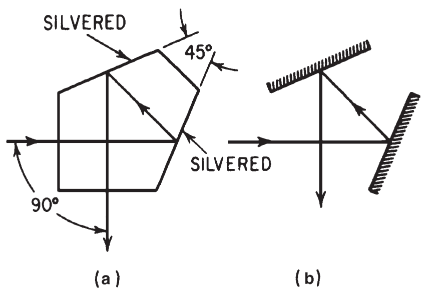

Figure 4.29 The Penta prism (a) and its equivalent mirror system (b).

The Penta prism is used where it is desirable to produce an exact 90° deviation without having to orient the prism precisely. The end reflectors of rangefinders are often of this type, and in optical tooling and precise alignment work, the Penta prism is useful to establish an exact 90° angle. In large rangefinders, however, the prism is replaced by two mirrors (Fig. 4.29b), securely cemented to a block in order to avoid the weight, absorption, and cost of a large block of solid glass.

Occasionally a roof is substituted for one of the reflecting faces of the Penta prism to invert the image in one meridian.

4.14 Rhomboids and Beam Splitters

The rhomboid prism is a simple means of displacing the line of sight without affecting the orientation of the image or deviating the line of sight. The rhomboid prism and its mirror system equivalent are shown in Fig. 4.30.

A beamsplitter is frequently useful for the purpose of combining two beams (or images) into one, or for separating one beam into two. A thin plate of glass with one surface coated with a semireflecting coating, as shown in Fig. 4.31a, can be used for this purpose, but it suffers from two drawbacks. First, if used in a convergent or divergent beam, it would introduce astigmatism, and second, the reflection from the second surface, although faint, would produce a ghost image displaced from the primary image. (Note that in parallel light neither of these objections is valid, provided the surfaces of the plate are accurately parallel.) The beamsplitter cube (Fig. 4.31b) avoids these difficulties. It is composed of two right-angle prisms cemented together. The hypotenuse of one prism is coated with a semireflecting coating before cementing.

Where the weight or absorption of the cube cannot be tolerated, a *pellicle* is often used as a semireflector. A pellicle is a thin (2- to 10-μm) membrane (usually a plastic such as nitrocellulose) stretched over a frame; by virtue of its extreme thinness, both the astigmatism and ghost displacement are reduced to acceptable values.

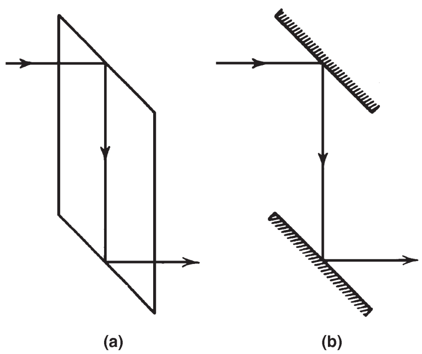

Figure 4.30 (a) Rhomboid prism. (b) An equivalent mirror system. Both systems displace the optical axis without deviation or reorientation of the image.

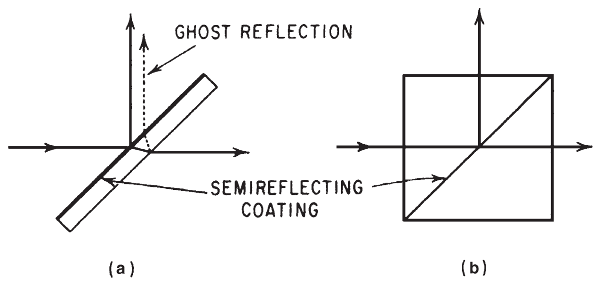

Figure 4.31 Beamsplitters. (a) A thin parallel plate is convenient but may be objectionable because of ghosting and astigmatism, unless used in parallel light. (b) Beamsplitting cube has a semireflecting coating supplied to one of the diagonal faces before cementing.

Obviously, the shape of the pellicle surface is determined by the shape of the frame over which it is stretched, and an accurately plane support is necessary. There are two less obvious features of the pellicle which may be disadvantageous: (1) Interference between light reflected from the two surfaces of the extremely thin pellicle can result in a transmission that varies in a rippled way as a function of wavelength, and (2) the pellicle can act as if it were the diaphragm of a microphone, and any atmospheric vibrations can change the shape of the reflecting surface, introducing significant changes in the imagery of the system. This is the basis for one "talk-on-a-beam-of-light" toy.

Figure 4.32 shows a prism which is often used in microscope eyepieces to change the direction of the line of sight from vertical to a more-convenient-to-use 45°. As shown, the prism can be used as a beamsplitter either to provide for coaxial illumination or to allow a second eyepiece; without the beamsplitting feature, it simply redirects the line of sight.

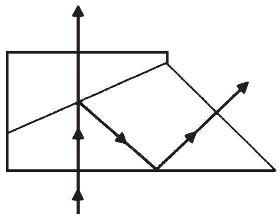

Figure 4.32

In Fig. 4.33, two binocular eyepiece prism systems are sketched. Both serve the same function, namely splitting the light beam from an objective lens into two parts. The two beams are displaced sufficiently so that they can be presented to two eyepieces and both eyes may simultaneously view the same subject. Notice that in both systems, extra glass has been added to the left-hand path so that the amount of glass in each path is identical; in this way the aberrations introduced by the glass are the same for each path. Most of the glass in these systems could be dispensed with if desired, since each of them is equivalent to a beamsplitting cube plus three reflectors. In the system shown in Fig. 4.33b, the two halves can be rotated about the objective axis to vary the spacing between the eyepieces as shown in Fig. 433c. Notice that the image is not rotated by this procedure but retains its original orientation, because the reflecting surfaces are in the form of a rhomboid prism.

Often two Porro systems are used in a rotatable configuration which allows a change in the eye separation.

4.15 Plane Mirrors

In the preceding discussions we have indicated several times that reflecting prisms may be replaced by mirrors. For most applications, it is necessary that the mirrors be first-surface mirrors, as opposed to ordinary second-surface mirrors. The two types are sketched in Fig. 4.34. The first-surface mirror is usually preferable because it does not produce a ghost image as does the second-surface mirror. In addition, the second-surface mirror requires the processing of an extra surface in its fabrication. It also requires the light to pass through a thickness of glass which may introduce aberrations and which will absorb energy in ultraviolet and infrared applications. The second-surface mirror can be made more durable, however, since its reflecting coating can be protected from the elements by electrodeposited copper and painted coverings. First-surface mirrors are usually made with vacuum-deposited aluminum films protected by a thin transparent overcoating of silicon monoxide or magnesium fluoride.

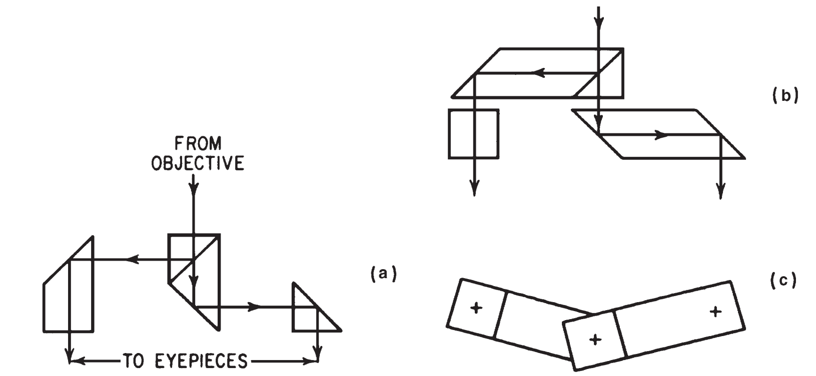

Figure 4.33 Prism systems for binocular eyepiece instruments. System (a) can be adjusted to match the user's eye separation by sliding both outer prisms in or out; this defocuses the instrument. Sketch (c) shows how the halves of (b) can be rotated about the objective axis to make this adjustment.

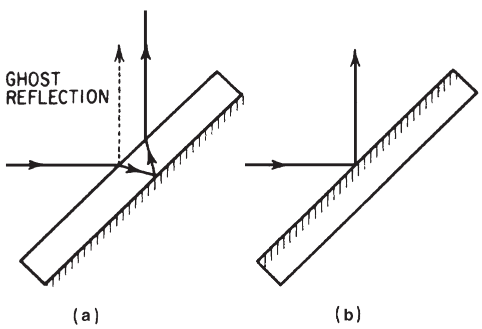

Figure 4.34 (a) Second-surface mirror. (b) First-surface mirror.

4.16 The Design of Prism and Reflector Systems

Ordinarily it is required of a prism (or reflector) system that it produce an image with a certain orientation and with the emergent beam of light redirected in a given manner. The design effort is usually best begun by establishing the minimum number of reflectors which will produce the desired result. This is most simply (and perhaps best) accomplished by straightforward trial and error. A rough perspective sketch is made to indicate the reflections necessary to locate the image in its desired position. The orientation of the image is then checked by the technique of Sec. 4.7; reflectors are added in various orientations until the image orientation is correct. Usually several roughly equivalent schemes are possible, and a selection can be made based on the requirements of the application.

When the reflection system is completed, the optical system is unfolded, i.e., sketched with the optical axis as a straight line. The object, image, and lens apertures are added to the sketch and the necessary sizes for the reflectors are determined in both meridians. If the system is to be composed of prisms, the unfolded layout is repeated with the axial distances adjusted to the "equivalent air thickness" (t/n) for that portion of the system which is glass so that the ray paths can be drawn as straight lines.

As an example of reflector system design, let us consider the problem presented by Fig. 4.35. The object at A is to be projected by an ordinary lens B onto a screen at S. The plane of S is parallel to the original projection axis and its center is above the axis by some amount Y. The required orientations of object and image are shown in the sketch.

We begin by noting that the image formed by the projection lens will be inverted in both meridians with respect to the object, as shown at C in Fig. 4.35. Now, passing to Fig. 4.36, let us consider the effect of a mirror placed at D. Of the four directions shown as possible reflections at D, the upward reflection labeled D_1 seems the most promising since it sends the light in a direction that it must eventually take, so we elect to pursue this line. Using similar reasoning at E, we should be inclined to select E_2; however, the image at E_2 is rotated 90° from our desired orientation. Selecting E_1 on the basis that its image orientation is closest to the desideratum, we consider a reflection at F. Again, F_3 is in the proper direction, but the image is reversed from left to right. Case F_1 has the proper orientation, but the light is traveling away from the screen. If we add a mirror to reverse the direction of propagation, we will have both orientation and direction as required. To accomplish this without directing the light back through F, we must resort to a figure 4 arrangement as shown in Fig. 4.37, which diagrams the entire system.

It is quite apparent that Fig. 4.37 represents only one of the many possible arrangements of mirrors which could be utilized to accomplish

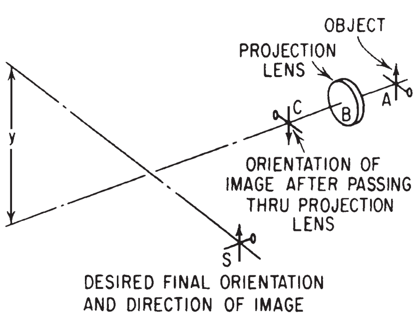

Figure 4.35

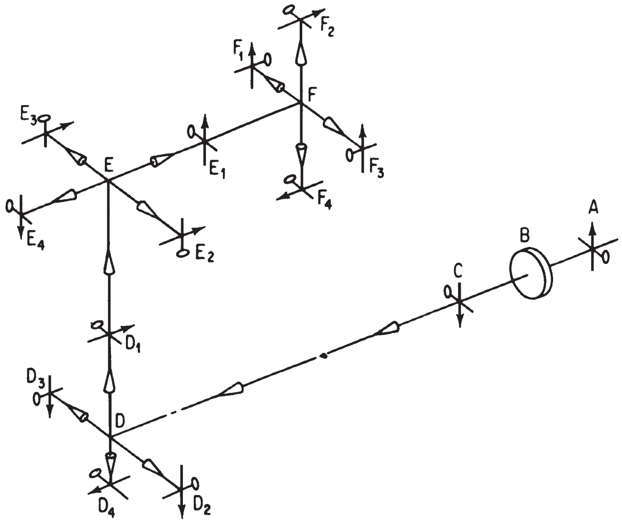

Figure 4.36

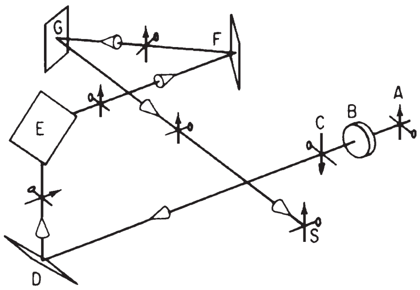

Figure 4.37

this same end result. The reader may also have noticed that the discussion has been limited to reflections for which the plane of incidence lay in one of the cartesian reference planes, and also that first consideration was given to reflections which deviated the axis by 90°. For the novice, these restrictions have much to recommend them; one is well advised to keep first trials of this type as simple and uncomplicated as possible. Further, the reduction of the system to practice is much simplified if compound angles are avoided. If our problem had required that the final image be rotated 45°, then we would necessarily have had to depart from the cartesian planes to achieve the desired result.

The Porro erecting prism (Fig. 4.38a) will serve as an illustrative example of the "unfolding" technique used in the design of prism systems. The prisms have been unfolded in Fig. 4.38b (for clarity, the second prism is shown rotated 90° about the axis). Each prism can be seen to be the equivalent of a glass block whose thickness is twice the size of its end face. Notice that the rays from the lens are refracted at each air-glass surface of the system and that the image has been displaced to the right by the prisms.

In Fig. 4.38c, the prisms are drawn with their "equivalent air thickness" as discussed in Section 4.8. This allows us to draw the (paraxial) light rays through the prism as straight lines, simplifying the construction considerably.

Now let us suppose that we are to design the minimum size Porro system for a 7 × 50 binocular. The objective lens has a focal length of 7 in, an aperture of 2 in, and is to cover a $^5/_8$-in-diameter field, as sketched in Fig. 4.39a. We first note that the proportions of face width to "equivalent air thickness" for each prism (Fig. 4.39a) are $A{:}2A/n = 1{:}2/n$, or, if we assume an index of 1.50, 3:4. We begin the design from the image and work toward the objective. Placing the exit face of the prism $^1/_2$ in

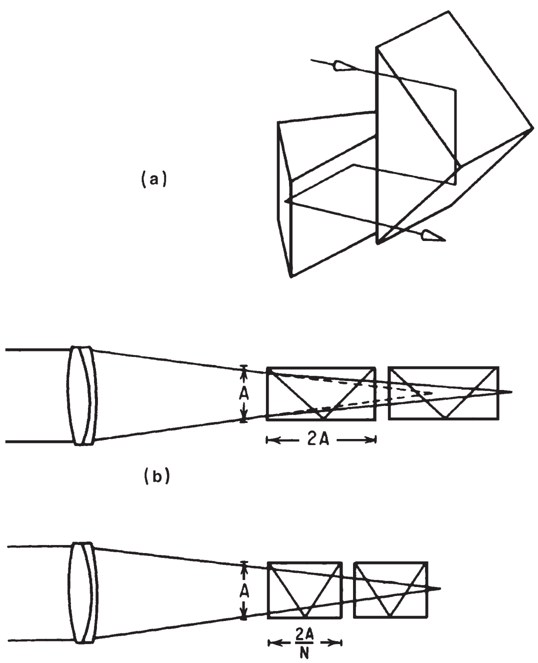

Figure 4.38 (a) Porro prism system (first type). (b) Unfolded prisms. Dashed lines indicate path rays would take without prisms. Solid line shows the displacement of the focal point by the prisms. (c) The prisms are drawn to their equivalent air thickness so that the rays can be drawn as straight lines.

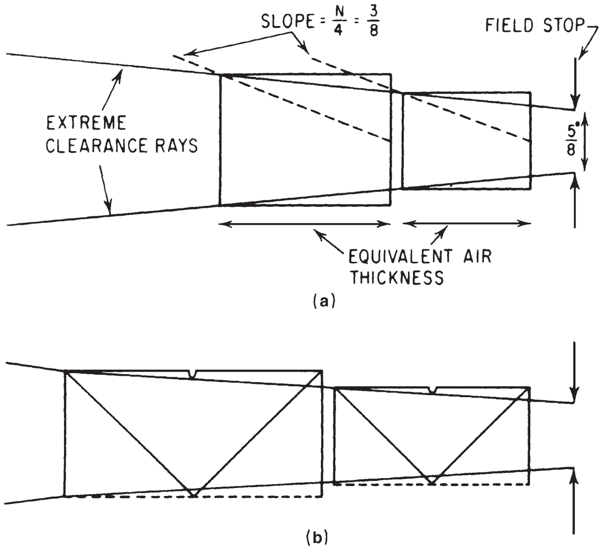

Figure 4.39 The layout of a minimum-size prism system is shown in (a). The extreme clearance rays connect the rim of the objective with the edge of the field of view. The intersection of the dashed lines (see text) with these rays locates the corner of the smallest prism which will pass the full image cone. In (b) the prisms are drawn to scale, showing their true thickness.

from the image (to allow for clearance and to keep the glass surface well out of the focal plane), we construct the dashed line shown in Fig. 4.39a with a slope of 3:8 (one-half the face-to-equivalent-thickness ratio) starting from the axial intercept of the exit face. This line is, of course, the locus of the corners of a family of prisms of various sizes, and the point where it intersects the extreme clearance ray defines the minimum size prism which will transmit the entire cone of light from the objective. For practical purposes, the prism should be made slightly larger than this to allow for bevels and mounting shoulders.

The procedure is now repeated for the other prism; an air space is left between the two to allow for the mounting plate to which both prisms are to be fastened. In Fig. 4.39b, the system is drawn to scale, with the prism blocks expanded to their true length. The reason for the ground slot usually cut into the hypotenuse faces of Porro prisms can be understood from an examination of the unfolded drawings. Light rays from outside the desired field of view can be reflected (by total internal reflection) from these faces back into the field where they are quite annoying; the slot intercepts these rays as they graze along the hypotenuse.

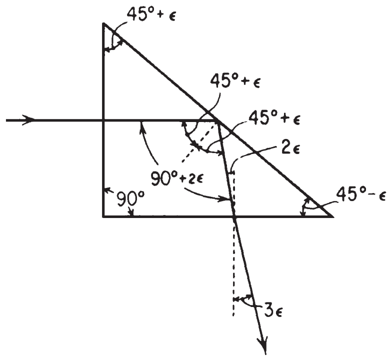

Figure 4.40 The passage of a ray through a right-angle prism whose hypotenuse face is tilted from its proper position by a small angle ϵ. After reflection, the ray is deviated by 2ϵ; this is increased to 3ϵ (or $2n\epsilon$) by refraction at the exit face.

4.17 Analysis of Fabrication Errors

The effects produced by errors in prism angles (due to manufacturing tolerances) are readily analyzed. Such angular errors can be treated as equivalent to the rotation of a reflecting surface from its nominal position, and/or the addition of a thin refracting prism to the system.

As an example, consider the right-angle prism shown in Fig. 4.40 and assume that the upper 45° angle is too large by ϵ and that the lower 45° angle is too small by ϵ. A ray normal to the entrance face will make an angle of incidence of $45° + \epsilon$ at the hypotenuse; the angle of reflection will then be $45° + \epsilon$ and the ray will be reflected through an angle of $90° + 2\epsilon$. Thus, rotating the reflecting face through ϵ has introduced an error of 2ϵ in the direction of the ray.

At the exit face, the ray has an angle of incidence of 2ϵ and, if the prism index is 1.5, an angle of refraction of 3ϵ. Thus, the total deviation of the ray from its nominal direction is 3ϵ. Also, since the ray has been deviated through an angle ϵ by refraction at this surface, the ray will be dispersed and spread out into a spectrum subtending an angle of ϵ/V according to Eq. 4.11.

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Note: Titles preceded by an asterisk are out of print.

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