## Chapter

## 1

# General Principles

### 1.1 The Electromagnetic Spectrum

This book deals with certain phenomena associated with a relatively narrow slice of the electromagnetic spectrum. Optics is often defined as being concerned with radiation visible to the human eye; however, in view of the recent expansion of optical applications in the regions of the spectrum on either side of the visible region, it seems not only prudent, but necessary, to include certain aspects of the infrared and ultraviolet regions in our discussions.

The known electromagnetic spectrum is diagramed in Fig. 1.1 and ranges from cosmic rays to radio waves. All the electromagnetic radiations transport energy and all have a common velocity in vacuum of $c = 2.998 \times 10^{10}$ cm/s. In other respects, however, the nature of the radiation varies widely, as might be expected from the tremendous range of wavelengths represented. At the short end of the spectrum we find gamma radiation with wavelengths extending below a billionth of a micron (one micron or micrometer = 1 $\mu$m = $10^{-6}$ m) and at the long end, radio waves with wavelengths measurable in miles. At the short end of the spectrum, electromagnetic radiation tends to be quite particlelike in its behavior, whereas toward the long wavelength end the behavior is mostly wavelike. Since the optical portion of the spectrum occupies an intermediate position, it is not surprising that optical radiation exhibits both wave and particle behavior.

The visible portion of this spectrum (Fig. 1.2) takes up less than one octave, ranging from violet light with a wavelength of 0.4 $\mu$m to red light with a wavelength of 0.76 $\mu$m. Beyond the red end of the spectrum lies the infrared region, which blends into the microwave region

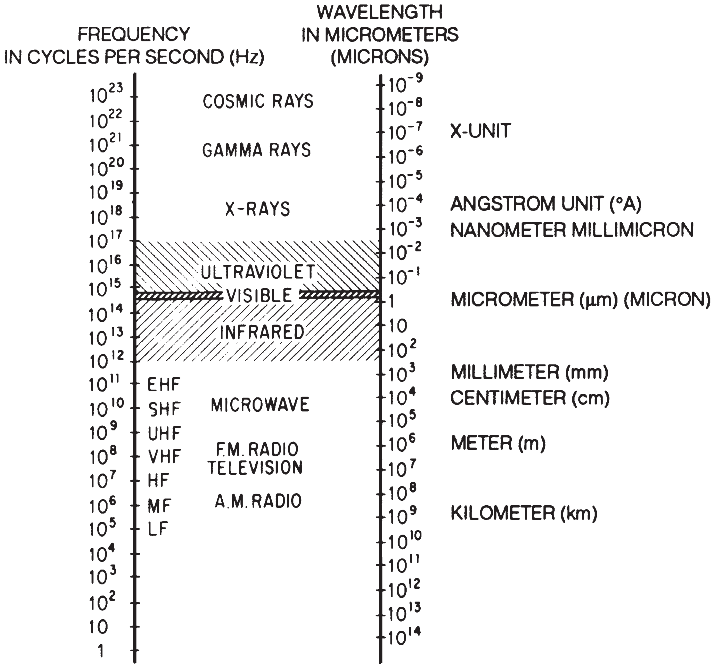

**Figure 1.1** The electromagnetic spectrum.

at a wavelength of about one millimeter. The ultraviolet region extends from the lower end of the visible spectrum to a wavelength of about 0.01 μm at the beginning of the x-ray region. The wavelengths associated with the colors seen by the eye are indicated in Fig. 1.2.

The ordinary units of wavelength measure in the optical region are the angstrom (Å); the millimicron (mμ), or nanometer (nm); and the micrometer (μm), or micron (μ). One micron is a millionth of a meter, a millimicron is a thousandth of a micron, and an angstrom is one ten-thousandth of a micron (see Table 1.1). Thus, $1.0\ \text{Å} = 0.1\ \text{nm} = 10^{-4}\ \mu\text{m}$. The frequency equals the velocity $c$ divided by the wavelength, and the wavenumber is the reciprocal of the wavelength, with the usual dimension of $\text{cm}^{-1}$.

## 1.2 Light Wave Propagation

If we consider light waves radiating from a point source in a vacuum as shown in Fig. 1.3, it is apparent that at a given instant each wave front is spherical in shape, with the curvature (reciprocal of the radius) decreasing as the wave front travels away from the point source. At a sufficient distance from the source the radius of the wave front may be regarded as infinite. Such a wave front is called a plane wave.

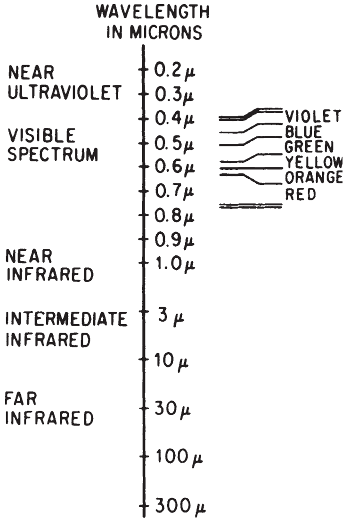

**Figure 1.2** The "optical" portion of the electromagnetic spectrum.

**TABLE 1.1 Commonly Used Wavelength Units**

| | | | | |
|---|---|---|---|---|
| Centimeter | = | $10^{-2}$ meter | | |
| Millimeter | = | $10^{-3}$ meter | | |
| Micrometer | = | $10^{-6}$ meter | = | $10^{-3}$ millimeter |
| Micron | = | $10^{-6}$ meter | = | $10^{-3}$ millimeter |
| Millimicron | = | $10^{-3}$ micron | = | 1.0 nanometer |
| | = | $10^{-6}$ millimeter | | |
| | = | $10^{-9}$ meter | | |
| Nanometer | = | $10^{-9}$ meter | = | 1.0 millimicron |
| Angstrom | = | $10^{-10}$ meter | = | 0.1 nanometer |

The distance between successive waves is of course the wavelength of the radiation. The velocity of propagation of light waves in vacuum is approximately $3 \times 10^{10}$ cm/s. In other media the velocity is less than in vacuum. In ordinary glass, for example, the velocity is about two-thirds of the velocity in free space. The ratio of the velocity in vacuum to the velocity in a medium is called the index of refraction of that medium, denoted by the letter $n$.

$$\text{Index of refraction } n = \frac{\text{velocity in vacuum}}{\text{velocity in medium}} \tag{1.1}$$

Both wavelength and velocity are reduced by a factor of the index; the frequency remains constant.

Ordinary air has an index of refraction of about 1.0003, and since almost all optical work (including measurement of the index of refraction) is carried out in a normal atmosphere, it is a highly convenient convention to express the index of a material relative to that of air (rather than vacuum), which is then assumed to have an index of exactly 1.0.

The actual index of refraction for air at 15°C is given by

$$(n-1) \times 10^8 = 8342.1 + \frac{2{,}406{,}030}{(130-\nu^2)} + \frac{15{,}996}{(38.9-\nu^2)}$$

where $\nu = 1/\lambda$ ($\lambda$ = wavelength, in μm). At other temperatures the index may be calculated from

$$(n_t-1) = \frac{1.0549\,(n_{15^\circ} - 1)}{(1 + 0.00366t)}$$

The change in index with pressure is 0.0003 per 15 lb/in$^2$, or 0.00002/psi.

If we trace the path of a hypothetical point on the surface of a wave front as it moves through space, we see that the point progresses as a straight line. The path of the point is thus what is called a ray of light. Such a light ray is an extremely convenient fiction, of great utility in understanding and analyzing the action of optical systems, and we shall devote the greater portion of this volume to the study of light rays. Note that the ray is normal to the wave front, and vice versa.

The preceding discussion of wave fronts has assumed that the light waves were in a vacuum, and of course that the vacuum was isotropic, i.e., of uniform index in all directions. Several optical crystals are anisotropic; in such media wave fronts as sketched in Fig. 1.3 are not spherical. The waves travel at different velocities in different directions, and thus at a given instant a wave in one direction will be further from the source than will a wave traveling in a direction for which the media has a larger index of refraction.

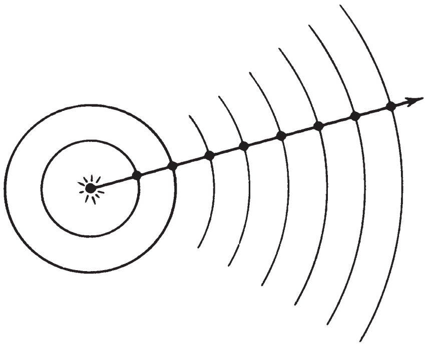

**Figure 1.3** Light waves radiating from a point source in an isotropic medium take a spherical form; the radius of curvature of the wave front is equal to the distance from the point source. The path of a point on the wave front is called a light ray, and in an isotropic medium is a straight line. Note also that the ray is normal to the wave front.

Although most optical materials may be assumed to be isotropic, with a completely homogeneous index of refraction, there are some significant exceptions. The earth's atmosphere at any given elevation is quite uniform in index, but when considered over a large range of altitudes, the index varies from about 1.0003 at sea level to 1.0 at extreme altitudes. Therefore, light rays passing through the atmosphere do not travel in exactly straight lines; they are refracted to curve toward the earth, i.e., toward the higher index. Gradient index optical glasses are deliberately fabricated to bend light rays in controlled curved paths. We shall assume homogeneous media unless specifically stated otherwise.

## 1.3 Snell's Law of Refraction

Let us now consider a plane wave front incident upon a plane surface separating two media, as shown in Fig. 1.4. The light is progressing from the top of the figure downward and approaches the boundary surface at an angle. The parallel lines represent the positions of a wave front at regular intervals of time. The index of the upper medium we shall call $n_1$ and that of the lower $n_2$. From Eq. 1.1, we find that the velocity in the upper medium is given by $v_1 = c/n_1$ (where $c$ is the velocity in vacuum $\approx 3 \times 10^{10}$ cm/s) and in the lower by $v_2 = c/n_2$. Thus, the velocity in the upper medium is $n_2/n_1$ times the velocity in the lower, and the distance which the wave front travels in a given interval of time in the upper medium will also be $n_2/n_1$ times that in the lower. In Fig. 1.4 the index of the lower medium is assumed to be larger so that the velocity in the lower medium is less than that in the upper medium.

At time $t_0$ our wave front intersects the boundary at point $A$; at time $t_1 = t_0 + \Delta t$ it intersects the boundary at $B$. During this time it has moved a distance

$$d_1 = v_1 \, \Delta t = \frac{c}{n_1} \, \Delta t \tag{1.2a}$$

in the upper medium, and a distance

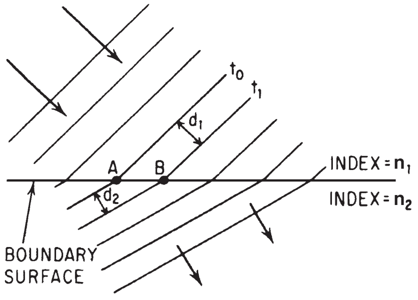

**Figure 1.4** A plane wave front passing through the boundary between two media of differing indices of refraction ($n_2 > n_1$).

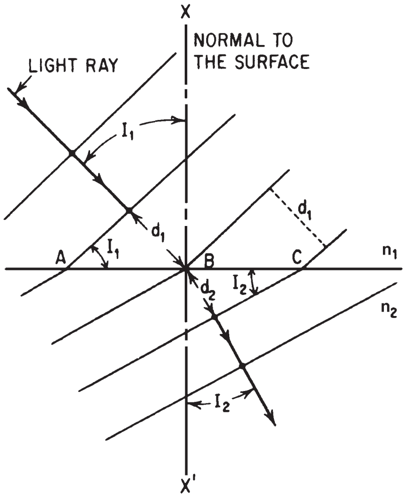

Figure 1.5

$$d_2 = v_2\,\Delta t = \frac{c}{n_2}\,\Delta t \qquad (1.2b)$$

in the lower medium.

In Fig. 1.5 we have added a ray to the wave diagram; this ray is the path of the point on the wave front which passes through point *B* on the surface and is normal to the wave front. If the lines represent the positions of the wave at equal intervals of time, *AB* and *BC,* the distances between intersections, must be equal. The angle between the wave front and the surface ($I_1$ or $I_2$) is equal to the angle between the ray (which is normal to the wave) and the normal to the surface *XX′*. Thus we have from Fig. 1.5

$$AB = \frac{d_1}{\sin I_1} = BC = \frac{d_2}{\sin I_2}$$

and if we substitute the values of $d_1$ and $d_2$ from Eq. 1.2, we get

$$\frac{c\ \Delta t}{n_1 \sin I_1} = \frac{c\ \Delta t}{n_2 \sin I_2}$$

which, after canceling and rearranging, yields

$$n_1 \sin I_1 = n_2 \sin I_2 \qquad (1.3)$$

This expression is the basic relationship by which the passage of light rays is traced through optical systems. It is called *Snell's law* after one of its discoverers.

Since Snell's law relates the sines of the angles between a light ray and the normal to the surface, it is readily applicable to surfaces other than the plane which we used in the example above; the path of a light ray may be calculated through any surface for which we can determine the point of intersection of the ray and the normal to the surface at that point.

The angle $I_1$ between the incident ray and surface normal is customarily referred to as the angle of incidence; the angle $I_2$ is called the angle of refraction.

For all optical media the index of refraction varies with the wavelength of light. In general the index is higher for short wavelengths than for long wavelengths. In the preceding discussion it has been assumed that the light incident on the refracting surface was monochromatic, i.e., composed of only one wavelength of light. Figure 1.6 shows a ray of white light broken into its various component wavelengths by refraction at a surface. Notice that the blue light ray is bent, or refracted, through a greater angle than is the ray of red light. This is because $n_2$ for blue light is larger than $n_2$ for red. Since $n_2 \sin I_2 = n_1 \sin I_1 =$ a constant in this case, it is apparent that if $n_2$ is larger for blue light than red, then $I_2$ must be smaller for blue than red. This variation in index with wavelength is called dispersion; when used as a differential it is written $dn$, otherwise dispersion is given by $\Delta n = n_{\lambda 1} - n_{\lambda 2}$, where $\lambda_1$ and $\lambda_2$ are the wavelengths of the two colors of light for which the dispersion is given. *Relative* dispersion is given by $\Delta n/(n - 1)$ and, in effect, expresses the “spread” of the colors of light as a fraction of the amount that light of a median wavelength is bent.

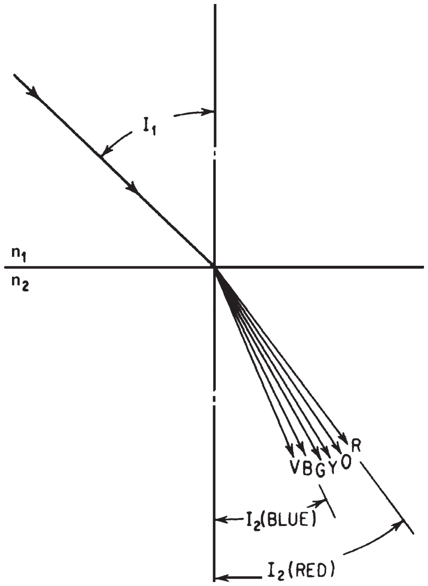

**Figure 1.6** Showing the dispersion of white light into its constituent colors by refraction (exaggerated for clarity).

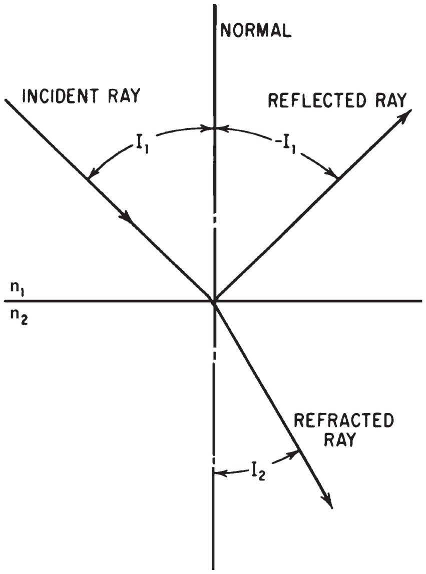

**Figure 1.7** Relationship between a ray incident on a plane surface and the reflected and refracted rays which result.

All of the light incident upon a boundary surface is not transmitted through the surface; some portion is reflected back into the incident medium. A construction similar to that used in Fig. 1.5 can be used to demonstrate that the angle between the surface normal and the reflected ray (the angle of reflection) is equal to the angle of incidence, and that the reflected ray is on the opposite side of the normal from the incident ray (as is the refracted ray). Thus, for reflection, Snell's law takes on the form

$$I_{\text{incident}} = -I_{\text{reflected}} \tag{1.4}$$

Figure 1.7 shows the relationship between a ray incident on a plane surface and the reflected and refracted rays which result.

At this point it should be emphasized that the incident ray, the normal, the reflected ray, and the refracted ray all lie in a common plane, called the plane of incidence, which in Fig. 1.7 is the plane of the paper.

## 1.4 The Action of Simple Lenses and Prisms on Wave Fronts

In Fig. 1.8 a point source $P$ is emitting light; as before, the arcs centered about $P$ represent the successive positions of a wave front at regular intervals of time. The wave front is incident on a biconvex lens consisting of two surfaces of rotation bounding a medium of (in this instance) higher index of refraction than the medium in which the source is located. In each interval of time the wave front may be assumed to travel a distance $d_1$ in the medium of the source; it will travel a lesser distance $d_2$ in the medium of the lens. (As in the preceding discussion, these distances are related by $n_1d_1 = n_2d_2$.) At some instant, the vertex of the wave front will just contact the vertex of the lens surface at point $A$. In the succeeding interval, the portion of the wave front inside the lens will move a distance $d_2$, while the portion of the same wave front still outside the lens will have moved $d_1$. As the wave front passes through the lens, this effect is repeated in reverse at the second surface. It can be seen that the wave front has been retarded by the medium of the lens and that this retardation has been greater in the thicker central portion of the lens, causing the curvature of the wave front to be reversed. At the left of the lens the light from $P$ was diverging, and to the right of the lens the light is now converging in the general direction of point $P'$. If a screen or sheet of paper were placed at $P'$, a concentration of light could be observed at this point. The lens is said to have formed an image of $P$ at $P'$. A lens of this type is called a converging, or positive, lens. The object and image are said to be *conjugates.*

Figure 1.8 diagrams the action of a convex lens—that is, a lens which is thicker at its center than at its edges. A convex lens with an index higher than that of the surrounding medium is a converging lens, in that it will increase the convergence (or reduce the divergence) of a wave front passing through it.

In Fig. 1.9 the action of a concave lens is sketched. In this case the lens is thicker at the edge and thus retards the wave front more at the edge than at the center and increases the divergence. After passing through the lens, the wave front appears to have originated from the neighborhood of point $P'$, which is the image of point $P$ formed by the lens. In this case, however, it would be futile to place a screen at $P'$ and

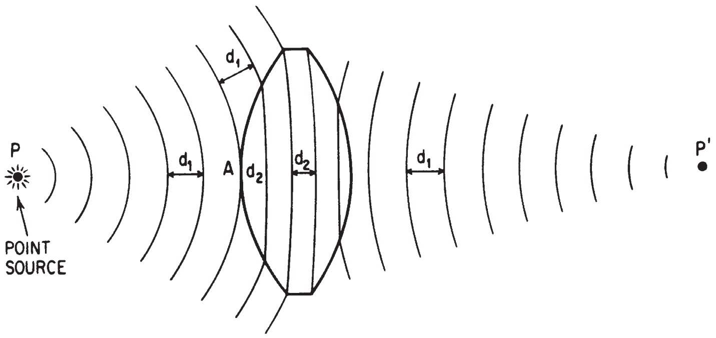

**Figure 1.8** The passage of a wave front through a converging, or positive, lens element.

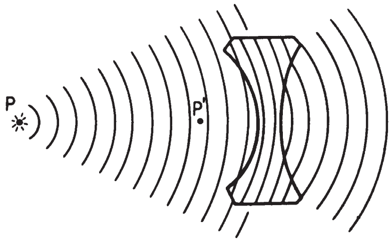

**Figure 1.9** The passage of a wave front through a diverging, or negative, lens element.

expect to find a concentration of light; all that would be observed would be the general illumination produced by the light emanating from *P*. This type of image is called a *virtual* image to distinguish it from the type of image diagramed in Fig. 1.8, which is called a *real* image. Thus a virtual image may be observed directly or may serve as a source to be reimaged by a subsequent lens system, but it cannot be produced on a screen. The terms "real" and "virtual" also may be applied to rays, where "virtual" applies to the extended part of a real ray.

The path of a *ray* of light through the lenses of Figs. 1.8 and 1.9 is the path traced by a point on the wave front. In Fig. 1.10 several ray paths have been drawn for the case of a converging lens. Note that the rays originate at point *P* and proceed in straight lines (since the media involved are isotropic) to the surface of the lens where they are refracted according to Snell's law (Eq. 1.3.) After refraction at the second surface the rays converge at the image $P'$. (In practice the rays will converge exactly at $P'$ only if the lens surfaces are suitably chosen surfaces of rotation, usually nonspherical, whose axes are coincident and pass through *P*.) This would lead one to expect that the concentration of light at $P'$ would be a perfect point. However, the wave nature of light causes it to be diffracted in passing through the limiting aperture of the lens so that the image, even for a "perfect" lens, is spread out into a small disc of light surrounded by faint rings as discussed in Chap. 6.

In Fig. 1.11 a wave front from a source so far distant that the curvature of the wave front is negligible is shown approaching a prism, which has two flat polished faces. As it passes through each face of the prism, the light is refracted downward so that the direction of propagation is deviated. The angle of deviation of the prism is the angle between the incident ray and the emergent ray. Note that the wave front remains plane as it passes through the prism.

If the radiation incident on the prism consisted of more than one wavelength, the shorter-wavelength radiation would be slowed down more by the medium composing the prism and thus deviated through a greater angle. This is one of the methods used to separate different wavelengths of light and is, of course, the basis for Isaac Newton's classic demonstration of the spectrum.

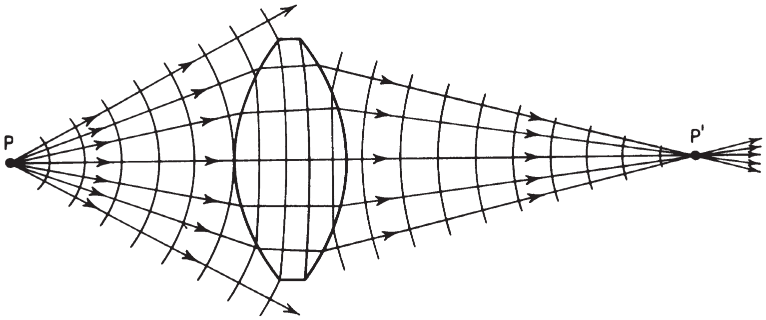

**Figure 1.10** Showing the relationship between light rays and the wave front in passing through a positive lens element.

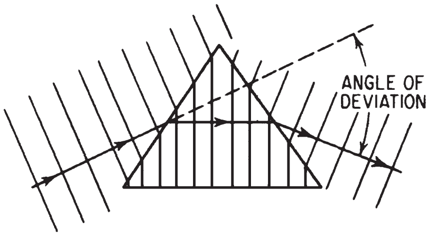

**Figure 1.11** The passage of a plane wave front through a refracting prism.

## 1.5 Interference and Diffraction

If a stone is dropped into still water, a series of concentric ripples, or waves, is generated and spreads outward over the surface of the water. If two stones are dropped some distance apart, a careful observer will notice that where the waves from the two sources meet there are areas with waves twice as large as the original waves and also areas which are almost free of waves. This is because the waves can reinforce or cancel out the action of each other. Thus if the crests (or troughs) of two waves arrive simultaneously at the same point, the crest (or trough) generated is the sum of the two wave actions. However, if the crest of one wave arrives at the same instant as the trough of the other, the result is a cancellation. A more spectacular display of wave reinforcement can often be seen along a sea wall where an ocean wave which has struck the wall and been reflected back out to sea will combine with the next incoming wave to produce an eruption where they meet.

Similar phenomena occur when light waves are made to interfere. In general, light from the same point on the source must be made to travel two separate paths and then be recombined, in order to produce optical interference. The familiar colors seen in soap bubbles or in oil films on wet pavements are produced by interference.

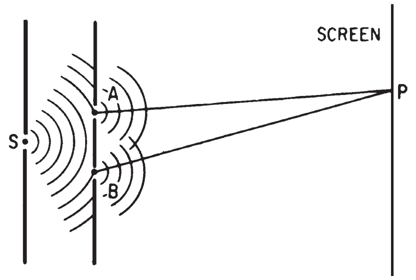

**Figure 1.12** Young's diffraction experiment.

*Young's experiment,* which is diagramed schematically in Fig. 1.12, illustrates both diffraction and interference. Light from a source to the left of the figure is caused to pass through a slit or pinhole *s* in an opaque screen. According to *Huygens' principle,* the propagation of a wave front can be constructed by considering each point on the wave front as a source of new spherical wavelets; the envelope of these new wavelets indicates the new position of the wave front. Thus *s* may be considered as the center of a new spherical or cylindrical wave (depending on whether *s* is a pinhole or a slit), provided that the size of *s* is sufficiently small. These diffracted wave fronts from *s* travel to a second opaque screen which has two slits (or pinholes), *A* and *B,* from which new wave fronts originate. The wave fronts again spread out by diffraction and fall on an observing screen some distance away.

Now, considering a specific point *P* on the screen, if the wave fronts arrive simultaneously (or in phase), they will reinforce each other and *P* will be illuminated. However, if the distances *AP* and *BP* are such that the waves arrive exactly out of phase, destructive interference will occur and *P* will be dark.

If we assume that *s, A,* and *B* are so arranged that a wave front from *s* arrives simultaneously at *A* and *B* (that is, distance *sA* exactly equals distance *sB*), then new wavelets will start out simultaneously from *A* and *B* toward the screen. Now if distance *AP* exactly equals distance *BP,* or if *AP* differs from *BP* by exactly an integral number of wavelengths, the wave fronts will arrive at *P* in phase and will reinforce. If *AP* and *BP* differ by one-half wavelength, then the wave actions from the two sources will cancel each other.

If the illuminating source is monochromatic, i.e., emits but a single wavelength of light, the result will be a series of alternating light and dark bands of gradually changing intensity on the screen (assuming that *s, A,* and *B* are slits), and by careful measurement of the geometry of the slits and the separation of the bands, the wavelength of the radiation may be computed. (The distance *AB* should be less than a millimeter and the distance from the slits to the screen should be to the order of a meter to conduct this experiment.)

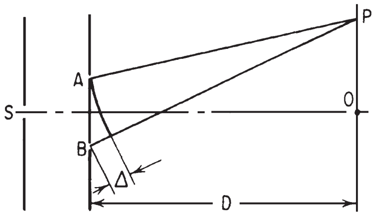

**Figure 1.13** Geometry of Young's experiment.

With reference to Fig. 1.13, it can be seen that, to a first approximation, the path difference between *AP* and *BP*, which we shall represent by Δ, is given by

$$\Delta = \frac{AB \cdot OP}{D}$$

Rearranging this expression, we get

$$OP = \frac{\Delta \cdot D}{AB} \tag{1.5}$$

Now as Fig. 1.13 is drawn, it is obvious that the optical paths *AO* and *BO* are identical, so the waves will reinforce at *O* and produce a bright band. If we set Δ in Eq. 1.5 equal to (plus or minus) one-half wavelength, we shall then get the value of *OP* for the first dark band

$$OP\ (\text{1st dark}) = \frac{\pm\lambda D}{2AB} \tag{1.6}$$

and if we assume that the distance from slits to screen *D* is one meter, that the slit separation *AB* is one-tenth of a millimeter, and that the illumination is red light of a wavelength of 0.64 μm, we get the following by substitution of these values in Eq. 1.6:

$$OP\ (\text{1st dark}) = \frac{\pm\lambda 10^3}{2 \cdot 10^{-1}} = \frac{\pm 10^4\lambda}{2} = \frac{\pm 10^4 \cdot 0.64 \cdot 10^{-3}}{2} = \pm 3.2 \text{ mm}$$

Thus the first dark band occurs 3.2 mm above and below the axis. Similarly the location of the next light band can be found at 6.4 mm by setting Δ equal to one wavelength, and so on.

If blue light of wavelength 0.4 μm were used in the experiment, we would find that the first dark band occurs at ±2 mm and the next bright band at ±4 mm.

Now if the light source, instead of being monochromatic, is white and consists of all wavelengths, it can be seen that each wavelength will produce its own array of light and dark bands of its own particular spacing. Under these conditions the center of the screen will be illuminated by all wavelengths and will be white. As we proceed from the center, the first effect perceptible to the eye will be the dark band for blue light which will occur at a point where the other wavelengths are still illuminating the screen. Similarly, the dark band for red light will occur where blue and other wavelengths are illuminating the screen. Thus a series of colored bands is produced, starting with white on axis and progressing through red, blue, green, orange, red, violet, green, and violet, as the path difference increases. Further from the axis, however, the various light and dark bands from all the visible wavelengths become so "scrambled" that the band structures blend together and disappear.

*Newton's rings* are produced by the interference of the light reflected from two surfaces which are close together. Figure 1.14 shows a beam of parallel light incident on a pair of partially reflecting surfaces. At some instant a wave front $AA'$ strikes the first surface at $A$. The point on the wave front at $A$ travels through the space between the two surfaces and strikes the second surface at $B$ where it is partially reflected; the reflected wave then travels upward to pass through the first surface again at $C$. Meanwhile the point on the wave front at $A'$ has been reflected at point $C$ and the two paths recombine at this point.

Now if the waves arrive at $C$ in phase, they will reinforce; if they arrive one-half wavelength out of phase, they will cancel. In determining the phase relationship at $C$ we must take into account the index of the material through which the light has traveled and also the phase change which occurs on reflection. This phase change occurs when light traveling through a low-index medium is reflected from the surface of a high-index medium; the phase is then abruptly changed by 180°, or one-half wavelength. No phase change occurs when the indices are encountered in reverse order. Thus with the relative indices as indicated in Fig. 1.14, there is a phase change at $C$ for the light following the $A'CD$ path, but no phase change at $B$ for the light reflected from the lower surface.

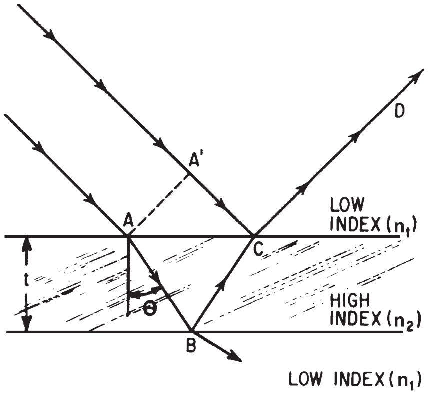

Figure 1.14

As in the case of Young's experiment described above, the difference between the optical paths $ABC$ and $A'$C determines the phase relationship. Since the index of refraction is inversely related to the velocity of light to a medium, it is apparent that the length of time a wave front takes to travel through a thickness $d$ of a material of index $n$ is given by $t = nd/c$ (where $c \approx 3 \cdot 10^{10}$ cm/s = velocity of light in vacuum). The constant frequency of electromagnetic radiation is given by $c/\lambda$, so that the number of cycles which take place during the time $t = nd/c$ is given by $(c/\lambda) \cdot (nd/c)$ or $nd/\lambda$. Thus, if the number of cycles are the same, or differ by an integral number of cycles, over the two paths of light traversed, the two beams of light will arrive at the same phase.

In Fig. 1.14, the number of cycles for the path $A'C$ is given by $^1/_2 + n_1A'C/\lambda$ (the one-half cycle is for the reflection phase change) and for the path $ABC$ by $n_2ABC/\lambda$; if these numbers differ by an integer, the waves will reinforce; if they differ by an integer plus one-half, they will cancel.

The use of cycles in this type of application is inconvenient, and it is customary to work in *optical path length,* which is the physical distance times the index and is a measure of the "travel time" for light. It is obvious that if we consider the difference between the two path lengths (arrived at by multiplying the above number of cycles by the wavelength $\lambda$), exactly equivalent results are obtained when the difference is an integral number of wavelengths (for reinforcement) or an integral number plus one-half wavelength (for cancellation). Thus, for Fig. 1.14, the *optical path difference* (OPD) is given by

$$\text{OPD} = \frac{\lambda}{2} + n_1A'C - n_2ABC \tag{1.7}$$

or

$$\text{OPD} = \frac{\lambda}{2} + 2n_2t\cos\theta$$

when the phase change is taken into account by the $\lambda/2$ term.

The term "Newton's rings" usually refers to the ring pattern of interference bands formed when two spherical surfaces are placed in intimate contact. Figure 1.15 shows the convex surface of a lens resting on a plane surface. At the point of contact the difference in the optical paths reflected from the upper and lower surfaces is patently zero. The phase change on reflection from the lower surface causes the beams to rejoin exactly out of phase, resulting in complete cancellation and the appearance of the central "Newton's black spot." Some distance from the center the surfaces will be separated by exactly one-quarter wavelength, and this path difference of one-half

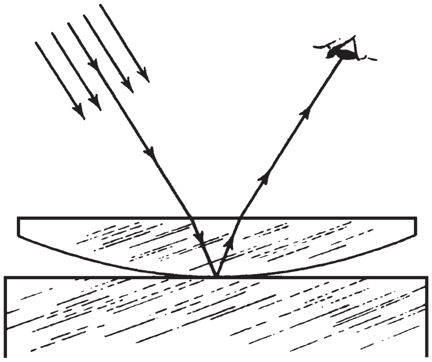

Figure 1.15

wavelength plus the phase change results in reinforcement, producing a bright ring. A little further from the center, the separation is one-half wavelength, resulting in a dark ring, and so on.

Just as in Young's experiment, the dark and bright bands for different wavelengths will occur at different distances from the center, resulting in colored circles near the point of contact which fade away toward the edge.

A setup similar to Fig. 1.15 can obviously be used to measure the wavelength of light if the radius of curvature of the lens is known and a careful measurement of the diameters of the light and dark fringes is made. The spacing between the surfaces is the sagittal height (SH) of the radius, given by

$$\mathrm{SH} = R - (R^2 - Y^2)^{1/2} \tag{1.8}$$

where $Y$ is the semidiameter of the ring measured. SH is equal to $\lambda/4$ for the first bright ring, $\lambda/2$ for the first dark ring, $3\lambda/4$ for the second bright ring, and so on.

## 1.6 The Photoelectric Effect

In the preceding section, the discussion was based upon the assumption that light was wavelike in nature. This assumption provides reasonable explanations for reflection, refraction, interference, diffraction, and dispersion, as well as other effects. The photoelectric effect, however, seems to require for its explanation that light behave as if it consisted of particles.

In brief, when short-wavelength light strikes a photoelectric material, it can knock electrons out of the material. As stated, this effect could be explained by the energy of the light waves exciting an electron sufficiently for it to break loose. However, when the nature of the incident radiation is modified, the characteristics of the emitted electrons change in an unexpected way. As the intensity of the light is increased, the number of electrons is increased just as might be expected. If the wavelength is increased, however, the maximum velocity of the electrons emitted is reduced; if the wavelength is increased beyond a certain value (this value is characteristic of the particular photoelectric material used), the maximum velocity drops to zero and no electrons are emitted, regardless of the intensity. The energy of a photon in electron volts is given by 1.24 divided by the wavelength in micrometers (microns).

Thus the energy necessary to break loose an electron is not stored up until enough is available (as one would expect of the wavelike behavior of light.) The situation here is more analogous to a shower of particles, some of which have enough energy to break an electron loose from the forces which bind it in place. Thus the particles of shorter wavelength have sufficient energy to release an electron. If the intensity of light is increased, the number of electrons released is increased and their velocity remains unchanged. The longer-wavelength particles do not have enough energy to knock electrons loose, and when the intensity of the long-wavelength light is increased, the effect is to increase the number of particles striking the surface, but each particle is still insufficiently powerful to release an electron from its bonds.

The apparent contradiction between the wave and particle behavior of light can be resolved by assuming that every "particle" has a wavelength associated with it which is inversely proportional to its momentum. This has proved true experimentally for electrons, protons, ions, atoms, and molecules; for example, an electron accelerated by an electric field of a few hundred volts has a wavelength of a few angstroms ($10^{-4}$ $\mu$m) associated with it. Reference to Fig. 1.1 indicates that this wavelength is characteristic of x-rays, and indeed, electrons of this wavelength are diffracted in the same patterns (by crystal lattices) as are x-rays.

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## Exercises

**1** What is the index of a medium in which light has a velocity of $2 \times 10^{10}$ cm/sc?

ANSWER: 1.5

**2** What is the velocity of light in water, $n = 1.33$?

ANSWER: $2.26 \cdot 10^{10}$ cm/s

**3** A ray of light makes an angle of 30° with the normal to a surface. Find the angle to the normal after refraction if:

- **(a)** the ray is in air and the other material is glass, $n = 1.5$.
- **(b)** the ray is in water and the other material is air.
- **(c)** the ray is in water and the other material is glass.

ANSWER: (a) 19.5°, (b) 41.7°, (c) 26.3°

**4** Two 6-in-diameter optical flats are contacted at one edge and separated by a piece of paper (0.003-in thick) at the opposite edge. When illuminated by light of 0.000020-in wavelength, how many fringes will be seen? Assume normal incidence.

ANSWER: 300 fringes

**5** In Exercise 4, if the space between the flats is filled with water ($n = 1.333$), how many fringes will be seen?

ANSWER: 400 fringes

**6** The convex surface of a lens is in contact with a flat plate of glass. If the radius of the surface is 20 in, at what diameter will the first dark ring be seen? The second? The third? What are the ring diameters if the radius is 200 in?

ANSWER: 0.040 in, 0.05657 in, 0.06928 in; 0.1265 in, 0.1789 in, 0.2191 in